

A new method for the analysis of simple and complex planar rapid movements

Wacef Guerfali *, Réjean Plamondon

Laboratoire Scribens, École Polytechnique de Montréal, Département de génie électrique et de génie informatique, C.P. 6079, Succursale Centre-Ville, Montréal (Québec) H3C 3A7, Canada

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Abstract

Recent developments in the field of simple human movement modelling provide new ways in which to view complete models for analysing and understanding complex movements. Based on a kinematic theory and a vectorial delta-lognormal model recently proposed by Plamondon (1993a, 1995a,b,c, 1998), a new method for exploring and understanding the inherent mechanisms that govern planar movement generation and predict human behaviour is presented here. This paper describes an approach for analysing simple as well as complex movements such as cursive handwriting. It highlights some difficulties encountered in the analysis of complex movements. Problems such as the development of robust approaches to solve the reverse engineering problem of automatic parameter extraction of a succession of time-overlapped nonlinear functions are discussed. The analysis of natural cursive handwriting shows many interesting properties of the model and proposes new ways to study perturbed movement phenomena. © 1998 Elsevier Science B.V. All rights reserved.

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1. Introduction

The recovery and analysis of action plans that result in complex movements pose some theoretical and practical difficulties for neuroscientific studies. One of the most crucial problems is to distinguish and decompose a complex movement into fundamental units or strokes, taking into account the time-overlapping effect that enables the production of fluent movements. Indeed, many questions arise in the attempt to determine the number of fundamental units involved in a complex movement, and in addressing the question of how to completely characterise each individual movement. Some studies have focused, for example, on how human subjects react to perturbed movements and how

they correct the resulting trajectory when the target moves during execution (Goodale et al., 1986; Pelisson et al., 1986; Martin and Prablanc, 1992). Other studies have tried to find fundamental laws which link movement time and movement precision (see Plamondon and Alimi, 1997 for a survey), which requires a precise measure of the characteristics of each single neuromotor command. Heuristics can be used to measure these characteristics for simple movements, but, when the movement units are partially hidden in a complex movement, most of the methods become unusable.

The kinematic theory recently proposed and briefly summarized in this paper (see Plamondon, 1993a,b, 1995a,b, 1996, 1997) suggests a formal description of simple movements. The general case of complex planar movements is considered as a vectorial overlapping of simple strokes. Each stroke is totally described by a set of parameters that characterises the movement, both in the static and in the kinematic domains.

* Corresponding author. Tel: +1 514 3404711 (extn 4675); fax: +1 514 3404147; e-mail: guerfali@scribens.polymtl.ca

The first part of this paper describes the key elements of the kinematic theory and the vectorial delta-lognormal model used for the generation and analysis of simple and complex planar movements. Among other things, the model explains how movements can be described in the static and in the kinematic domains.

The second part of the paper proposes a scheme for simple movement analysis. First, a brief overview of the nonlinear regression technique to solve the reverse engineering problem is presented. This problem can be stated as follows: How can we extract from real 2D signals the parameters of the model that best fit the observed data? Approaches to parameter estimation are proposed in the paper, with an optimisation process based on the Levenberg–Marquardt method (Marquardt, 1963).

The third part of the paper describes an approach to complex movement analysis. Complex movements are defined in this paper as multiple-stroke movements with time-overlapping effects. The method proposes heuristics to locate partially hidden strokes in complex movements, in order to distinguish the effect of each stroke, and applies nonlinear regression techniques to the optimisation process. The problem of finding the initial conditions required to ensure the convergence of these techniques is also discussed.

The last part of the paper presents an application of the proposed method for handwriting analysis. It shows how the extracted parameters can be used to represent, understand or regenerate complex movements. The model also proposes new ways to analyse perturbed movements and study the effects of some properties, such as starting time, reaction time and movement time, taking into account the overlapping effect of the individual strokes.

2. The kinematic theory of rapid human movements

The analysis of rapid human movements presented in this paper is based on the kinematic theory recently proposed by R. Plamondon (Plamondon, 1993a,b, 1995a,b, 1996, 1998). The kinematic theory is aimed mainly at understanding the generation and control of simple and complex human movements. The theory has been shown in the past few years to be one of the best and most complete approaches to describing the global properties of the neuromuscular networks involved in a synergistic action (Plamondon and Alimi, 1997; Plamondon, 1998). It proposes within a single framework some explanations about the emergence of the basic kinematic relationships and psychophysical laws that have been consistently reported in studies dealing with rapid human movements over the last century (Plamondon and Alimi, 1997).

2.1. Simple rapid human movements

According to the kinematic theory, simple human movements can be described in the velocity domain as the response of a synergistic action of an agonist and an antagonist neuromuscular network (Plamondon, 1993a, 1995a). Each network is composed of a large set of coupled neuromuscular subsystems that react to an input command D_1 (for the agonist) and D_2 (for the antagonist) with an impulse response that can be described by a lognormal function (Plamondon, 1993a, 1995a). Each lognormal impulse response $\Lambda(t; t_0, \mu_i, \sigma_i^2)$ can be characterised by three parameters: the starting time t_0 , the parameter μ_i which reflects its logtime delay, and σ_i^2 which reflects its logresponse time (Plamondon, 1993a, 1995a). The resulting curvilinear velocity $V(t)$ of a single movement is then described by subtracting the weighted impulse response of the antagonist network from the agonist one, which is called a delta-lognormal response (Eqs. (1) and (2)).

$$V(t) = D_1 \Lambda(t; t_0, \mu_1, \sigma_1^2) - D_2 \Lambda(t; t_0, \mu_2, \sigma_2^2) \quad (1)$$

$$\text{where } \Lambda(t; t_0, \mu_i, \sigma_i^2) = \frac{1}{\sqrt{2\pi\sigma_i}} e^{-\frac{1}{2\sigma_i^2}(\ln(t-t_0) - \mu_i)^2} \quad (2)$$

Eq. (1) results in velocity profiles that can have one, two or even three peaks (Plamondon, 1995a; Plamondon et al., 1993).

The vectorial delta-lognormal model (Plamondon, 1995c; Guerfali and Plamondon, 1995; Guerfali, 1996) describes single two-dimensional movements as a velocity vector, the magnitude of which follows a delta-lognormal response. Apart from the seven parameters of the delta-lognormal function, each velocity vector is also characterised in the space domain by three static parameters which globally reflect the geometric properties of the set of muscles and joints used in a particular movement: the starting point P_0 , the starting direction θ_0 and the global curvature C_0 (Plamondon and Guerfali, 1998). The curvature is considered to be positive if the movement is clockwise, and negative otherwise.

The angular direction of the velocity vector can then be deduced from the intrinsic relation that links the angular and the curvilinear velocities (Plamondon, 1987; Guerfali and Plamondon, 1995) (Eq. (3)).

$$\theta(t) = \theta_0 + C_0 \int_{t_0}^t V(\tau) d\tau \quad (3)$$

A single movement, also called a stroke (i), is thus represented in the space and velocity domains by a velocity vector starting at time $t_{0(i)}$ at point $P_{0(i)}$ with an initial direction $\theta_{0(i)}$, and moving along a circular path of length $D_{1(i)} - D_{2(i)}$ with a constant curvature $C_{0(i)}$. According to the kinematic theory, the movement described by this model will reach its target with a move-

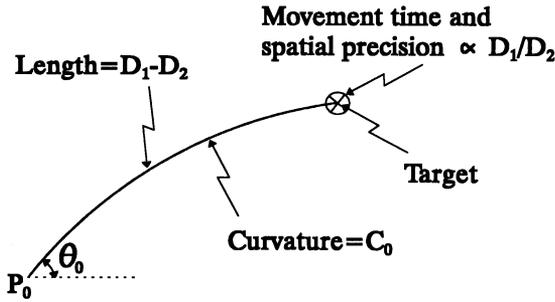


Fig. 1. A single movement or a stroke.

ment time that is proportional to the ratio of the agonist and antagonist commands, $D_{1(i)}/D_{2(i)}$ (Plamondon, 1995a,b) (see Fig. 1).

2.2. Two-dimensional complex rapid movements

As its name suggests, the vectorial delta-lognormal model considers a single movement as a vector moving along a circular path with a delta-lognormal velocity profile. Complex movements are then the result of the time-overlapping of two or more velocity vectors. The resulting velocity vector can be characterised by its instantaneous magnitude, which is the magnitude of the sum of individual velocity vectors describing each stroke (Eq. (4)) and its orientation (Eq. (5)).

$$V(t) = \left| \sum_{i=1}^n \vec{V}_{(i)}(t) \right| \tag{4}$$

$$\theta(t) = \arctan \left[\frac{\sum_{i=1}^n V_{(i)}(t) \sin(\theta_{(i)}(t))}{\sum_{i=1}^n V_{(i)}(t) \cos(\theta_{(i)}(t))} \right] \tag{5}$$

The analytical expression of angular velocity can be obtained by the time-derivation of Eq. (5). This results in a complex expression and no simple formulation is available (Plamondon and Guerfali, 1998). The vectorial delta-lognormal model not only explains the origin of the angular velocity signal, its shape and its properties, but also shows how this signal is not controlled independently, but emerges from the vectorial summation process (Guerfali, 1996).

Fig. 2 shows a simulated case of a two-stroke movement. The crosses in the space domain show the hidden path of each single stroke as taken separately, and the dots show the resulting path generated when the two movements overlap in time. In the velocity domain, we can see the bell-shaped velocity profiles for each single stroke (dotted lines), both in the angular and in the curvilinear domains, and then the result of the entire movement (solid lines). As can be seen, the shape of the angular velocity is influenced mainly by the angular discontinuity between consecutive strokes (Guerfali, 1996) as well as the time-overlapping of their curvilinear velocity (Plamondon and Guerfali, 1998). It can be shown by computer simulation that the curvilinear and the angular velocity are strongly linked, as observed in real movements. One should also note that the shape of the angular velocity is similar to that of the curvilinear velocity for each individual stroke taken separately (dotted lines). This similarity between the two signals is due to the simple relationship that exists between curvilinear and angular velocities when curvature is constant (see Eq. (6)). For real cases, this similarity is difficult to observe due to the discontinuity of the angular velocity at the starting and ending points where the curvilinear velocity is near 0 and the curvature is infinite.

$$V_{\theta(i)}(t) = C_{0(i)} \times V_{(i)}(t) \tag{6}$$

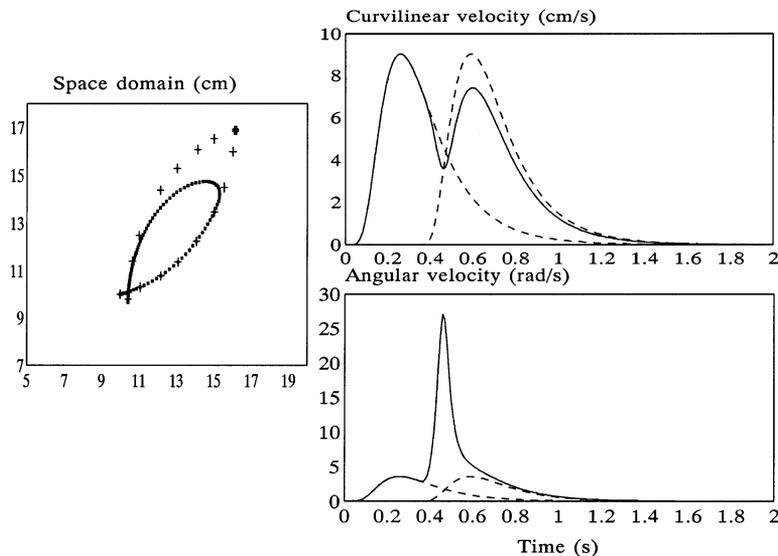


Fig. 2. Simulated overlapping strokes: generated trajectory, curvilinear and angular velocities.

This does explain, however, why the delta-lognormal equation is also successful in describing angular velocity data (Plamondon, 1995c, 1998) as collected in numerous experiments dealing with simple rotation, like wrist flexion or extension.

For complex movements (more than one stroke), the relationship between angular and curvilinear velocities becomes more complex, although the resulting curvature is not necessarily constant along the trajectory, but a function of time $C(t)$. We can also see in Fig. 2 that the maximum of the angular velocity corresponds to the minimum of the curvilinear velocity, which explains the phase shift observed in real movements between these two signals.

3. Simple movement analysis

To understand, represent and reproduce human movements, an analysis-by-synthesis approach is necessary to extract the set of vectorial delta-lognormal model parameters that best fit the movements under study. The extracted parameters then reflect the command amplitude, the movement time and spatial precision, the temporal properties of the neuromuscular networks involved and the geometric characteristics of the movement analysed.

The extraction of the set of parameters that best fits a simple movement i (or one stroke movement) can be made in two steps: The first is to estimate the parameters that describe the kinematics of that movement $D_{1(i)}$, $D_{2(i)}$, $\mu_{1(i)}$, $\mu_{2(i)}$, $\sigma_{1(i)}$, $\sigma_{2(i)}$ and $t_{0(i)}$; and the second is to estimate the static parameters $C_{0(i)}$ and $\theta_{0(i)}$ that describe the geometric properties of the movement in the 2D plane. With this set of nine parameters (and knowing the starting point P_0), a simple movement can be completely characterised, represented and reproduced in both the kinematic and the spatial (or stroke) domains.

Analysing the delta-lognormal velocity profile, seeking the optimal set of parameters that best fits the movement observed, requires the use of some robust optimisation approaches that ensure algorithm convergence. Knowing that the delta-lognormal function (as well as the lognormal function) is nonlinear with respect to most of its parameters, nonlinear regression techniques are required to extract parameters from velocity signals. Several methods exist to solve the nonlinear regression problem (Bard, 1974), and only a brief overview of one of the most robust techniques used is presented here.

3.1. Nonlinear regression technique

The regression problem can be summarised as follows: Suppose that we have n measures (y_1, y_2, \dots, y_n) of

a dependent variable Y (the curvilinear velocity signal in our case), which depends on k independent variables (X_1, X_2, \dots, X_k) (the curvilinear velocity is only a function of the time t in our case). The relationship between the variable Y and the independent variables X_i is determined by a class of functions (a delta-lognormal function in our case), which depends on p parameters ($D_1, D_2, \mu_1, \mu_2, \sigma_1, \sigma_2, t_0, C_0$ and θ_0). Regression techniques look for the set of function parameters that best fits the measures according to some criteria (like the least-squares fit, for example). If the class of functions is nonlinear according to some of its parameters, and no easy transformation is known to linearise the function (which is the case for the delta-lognormal function), nonlinear regression techniques have to be used. In this case, no analytical solution is available, and an iterative process is needed to optimise the search for a better solution around a set of initial conditions or a starting point of the parameter space. The choice of this starting point becomes critical and determines the region of the final solution. Assuming the derivability of the function used with respect to all its parameters, robust methods, such as the Levenberg–Marquardt techniques, can be used (Marquardt, 1963). An iterative process is then required until the error becomes smaller than a certain threshold, or a certain amount of computation time has elapsed.

3.2. Estimation of the spatial parameters $C_{0(i)}$ and $\theta_{0(i)}$

Curvature can be estimated by different methods. One of the simplest, and one which can be used for both simple and complex movements, is the ratio between the angular and the curvilinear velocities (Eq. (7)).

$$C_{0(i)} \approx \frac{V_\theta(t_{\max(i)})}{V(t_{\max(i)})} \quad (7)$$

Velocity values can be measured around the maximum values of the curvilinear velocity (at time $t_{\max(i)}$) to reduce border and overlapping effects. Even if we cannot guarantee that, at time $t_{\max(i)}$, the overlapping is minimal, the relative influence of an error on the result will be minimal for the higher magnitude region of the curvilinear velocity. This simple relationship (Eq. (7)) can be very helpful, but must be taken with care if the overlapping effect of adjacent strokes becomes significant. For real signals, the value of the estimated curvature will be quite different, depending on the time chosen for the estimation ($t = t_{\max(i)} \pm \Delta t$). In practical situations, an average value between the curvatures computed for a few points around $t_{\max(i)}$ constitutes a better approximation than Eq. (7).

For an individual stroke i , the parameter $\theta_{0(i)}$ can also be estimated, for the same reasons that are de-

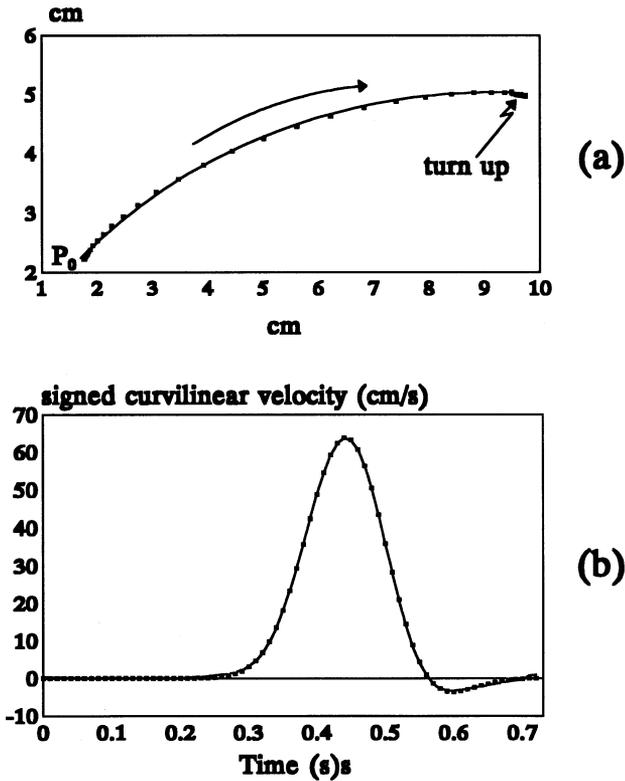


Fig. 3. One-stroke movement, original (solid lines) and generated after extraction (squares).

scribed above, around the point where the borders and the overlapping effects are minimised ($t_{\max(i)}$). Knowing the instantaneous angle $\theta(t)$ at time $t_{\max(i)}$, the previous estimation of the global curvature $C_{0(i)}$ and the initial estimation of the parameters ($D_{1(i)}$, $D_{2(i)}$, $\mu_{1(i)}$, $\mu_{2(i)}$, $\sigma_{1(i)}$, $\sigma_{2(i)}$ and $t_{0(i)}$), the initial orientation of each stroke can be evaluated by Eq. (8).

$$\theta_{0(i)} \approx \theta(t_{m(i)}) - C_{0(i)} \int_{t_{0(i)}}^{t_{m(i)}} V_{(i)}(t) dt \quad (8)$$

Fig. 3 shows a one-stroke movement (solid line) as reproduced by the vectorial delta-lognormal model (squares) after parameter estimation. The curvilinear velocity has been signed here (positive and negative parts) to highlight the turned-up portion of the stroke (or overshoot) at the end of the movement. This figure provides a typical example of a single movement, within multiple-peak velocity (2 here), which can be reproduced with a single set of parameters. Many of the models published so far (see Plamondon et al., 1993 and Alimi and Plamondon, 1993 for comparative reviews) would require a new set of parameters to reproduce the second peak.

4. Complex movement analysis

The analysis of complex movements (or multiple-stroke movements) poses more difficulties than that of simple movements. These problems are due to the fact that complex movements are mainly composed of time-overlapped strokes. The analysis of complex movements first requires the estimation of the minimal number of strokes that can generate the complex movement observed (or the minimal number of delta-lognormal curves that compose the curvilinear velocity signal). The second difficulty in analysing complex movements is due to time-overlapping effects. A simultaneous optimisation of all the parameters of the n strokes is needed. Each stroke i is described by a set of nine parameters ($D_{1(i)}$, $D_{2(i)}$, $\mu_{1(i)}$, $\mu_{2(i)}$, $\sigma_{1(i)}$, $\sigma_{2(i)}$, $t_{0(i)}$, $C_{0(i)}$ and $\theta_{0(i)}$), starting at point $P_{0(i)}$, where $P_{0(i)}$ is assumed to be the target of stroke $i-1$, except for the first stroke. The complexity of this problem increases rapidly with the number of strokes n , and convergence problems often become serious.

Development of robust parameter extraction techniques for the general problem of multiple-stroke extraction presents several difficulties. One of the major ones is the difficulty of developing approaches which make it possible to ensure the convergence of numerical nonlinear regression techniques when there is a large number of parameters to estimate. The numerical methods generally used for this type of problem and described above require a judicious choice of initial conditions. This choice becomes critical as the number of parameters increases. We present in the following subsection a heuristic method which partly solves these problems.

4.1. Localising partially hidden strokes

Estimating the number of partially hidden strokes and localising them can be achieved by inspection of the curvilinear velocity signal. First, each positive peak is considered as a potential velocity maximum of a hidden stroke. Second, in cases where more than one inflection point is detected between two successive maxima, a second-level analysis is required to determine the number of partially hidden strokes and approximate their location. Fig. 4 shows a typical curvilinear velocity signal where none, one, two or three inflection points are observed between two successive velocity maxima. In the case where two or more inflection points are detected in the velocity signal, one supplementary stroke is assumed between the two velocity maxima for further optimisation.

4.2. Choice of initial conditions

Estimating the initial conditions for the optimisation

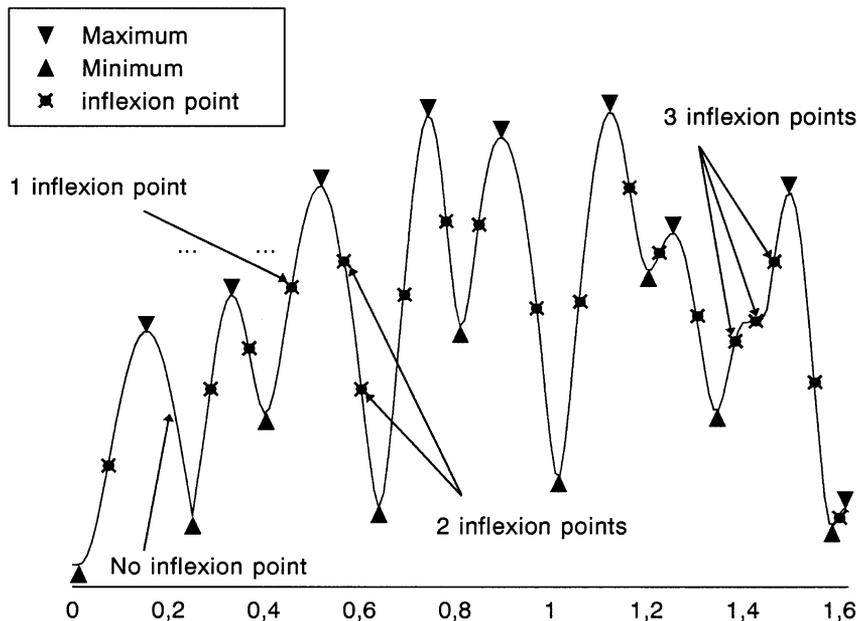


Fig. 4. Locating partially hidden strokes by inflexion point analysis.

method is a critical step, and a description of a heuristic iterative approach for this particular problem is presented here. First, the parameters of each delta-lognormal curve are roughly estimated by a heuristic or a local regression analysis. Second, the estimated strokes are iteratively extracted from the global velocity signal to eliminate, as much as possible, the effect of strokes overlapping within the rest of the signal. This step is repeated until the deviation between the old and the new estimates for each stroke taken individually is below a certain threshold. In practice, two to three iterations often prove to be sufficient. The algorithm used for this step of the processing can be summarised as follows:

/* Estimate the vectorial delta-lognormal parameters */

Initialise all the n delta-lognormal parameters to unknown

REPEAT the process X times (between 1 to 5)

Initialise: Working Signal = Original – Estimations

FOR each stroke i DO

Add the estimation of the i^{th} stroke

Estimate the i^{th} stroke within the region $[a, b]_n$ (see Fig. 5)

Subtract the new estimation from the i^{th} stroke

END (FOR)

END (REPEAT)

Fig. 5 illustrates how superimposition effects can be reduced using this algorithm for the first estimation step of the second stroke ($i = 2$). This figure shows how we can isolate the second stroke from the entire signal for the optimisation process (in this case estimating the

parameters of the second delta-lognormal curve V_2), knowing roughly the parameters of the adjacent strokes.

The first iteration that leads to a rough estimation of the parameters of each delta-lognormal curve uses a heuristic approach. A graphical method, described previously by Wise (1966) and adapted by Guerfali and Plamondon (1994), which enables estimation of the parameters of a lognormal curve, is applied. The parameters $\mu_{1(i)}$, $\mu_{2(i)}$, $\sigma_{1(i)}$ and $\sigma_{2(i)}$ are first assumed to be equal for the agonist and antagonist systems ($\mu_{1(i)} = \mu_{2(i)}$ and $\sigma_{1(i)} = \sigma_{2(i)}$), except that the area under the curve $D_{(i)}$ is divided between them in such a way that $D_{1(i)} > D_{2(i)}$ and $D_{(i)} = D_{1(i)} - D_{2(i)}$. During the second iteration, the parameters of each delta-lognormal curve taken separately are evaluated with a greater accuracy (using the nonlinear regression technique described above), since the adjacency effect can be subtracted from the rest of the signal. The second pass, which makes a better evaluation of the parameters, is more accurate if all the parameters are now set free to vary ($\sigma_{1(i)} \neq \sigma_{2(i)}$ and $\mu_{1(i)} \neq \mu_{2(i)}$, which was not the case at the first step). The regression technique used is still the Levenberg–Marquardt nonlinear regression method (Marquardt, 1963). The entire process is repeated for the desired number of iterations. Usually, two to three iterations are sufficient.

It might happen that parameter estimation for a particular stroke cannot be performed, particularly during the first iteration, because of extensive superimposition involving two or more adjacent strokes. In this

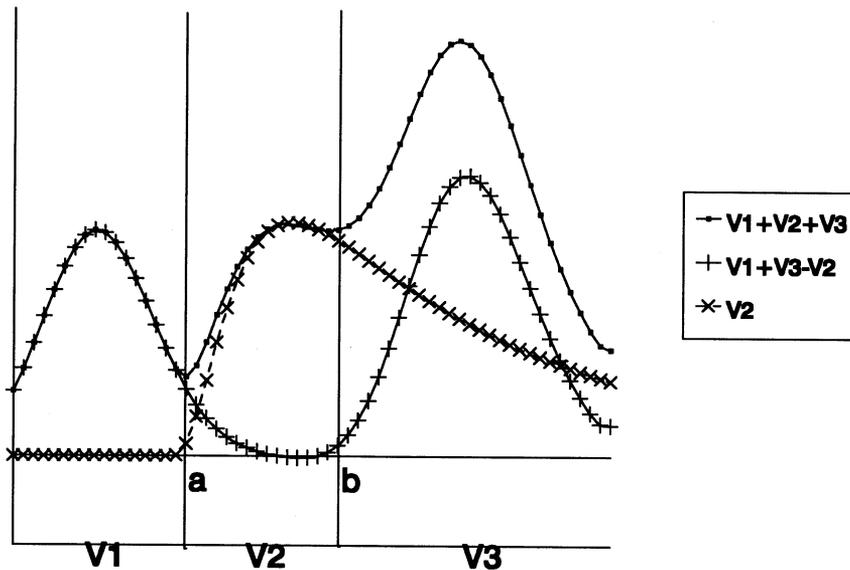


Fig. 5. Extraction of a partially hidden stroke from a complex movement.

case, the estimation of the delta-lognormal parameters is set aside and the next strokes are estimated. Estimation of the preceding strokes becomes possible once their immediate left and right neighbours have been estimated.

Fig. 6 shows a multiple-stroke movement (solid line) as reproduced by the vectorial delta-lognormal model (square dots) with the parameter estimation technique described above. As one can see, the parameters extracted enable a good reconstruction of the movement in both kinematic (curvilinear and angular velocities) and static (displacement path) domains.

As can be seen, the optimisation of the parameters that best fit both the kinematic and static domains is far from obvious, and the approach proposed here provides one possible way of doing it. Fig. 6 reveals that the results are very good, but still not perfect. However, it should be noted that, to our knowledge, this is the first time that a movement generation model has been used to fit experimental results in both the kinematic and static domains with such a good performance.

5. Application: Analysis of handwriting movements

Traditionally, handwriting analysis has been limited to direct measurements (X and Y sampled at a fixed frequency) from digitising tablets (Marquardt and Mai, 1994) from which velocity and acceleration signals were computed. Handwriting analysis was mainly limited by the fact that stroke-overlapping effects hide large parts of individual movements that compose a fluent complex path. Problems such as segmentation, movement time and movement composition cannot be analysed if we do not take into consideration the overlapping effects

of single movement units. The overlapping effect of strokes is observed each time a particular movement (i) starts before that movement ($i-1$) hits its final target. As can be seen in Fig. 7, the starting time of the second stroke plays a determinant role in the final shape of complex movements. Very different shapes can be generated with the same basic movements by changing only the starting time of the second stroke. If we consider each basic movement individually, or if we choose starting times in a way that minimises the overlapping effect (Fig. 7a), each stroke (i) will hit a target at a distance $D_{(i)}$ along its trajectory (where $D_{(i)} = D_{1(i)} - D_{2(i)}$), with spatial precision proportional to the ratio of $D_{1(i)}/D_{2(i)}$ (Plamondon, 1993a, 1995a). Fig. 7b,c shows simulated samples of the effect of varying the starting time of the second stroke $t_{0(2)}$ on the resulting movement. Small crosses show the underlying single movements, while the solid line shows the resulting trajectory. Analysis of the parameters extracted from the model can show, in those cases, the underlying strokes that would not otherwise be observed if only the resulting trajectory was studied.

With the overlapping of more than two simple movements, the relative starting time of each stroke (except for the first movement) will have an important influence on the shape of the resulting trajectory and will produce almost fluent movements. Fig. 8 shows two examples of the effect of the time-overlapping of multiple-stroke movements on the resulting path. Here again, the small crosses show the path of each individual stroke if no superimposition was observed, while the solid line shows the resulting path with stroke overlap. Another important property of the vectorial delta-lognormal model can be mentioned here: the final target for a complex movement is reached indepen-

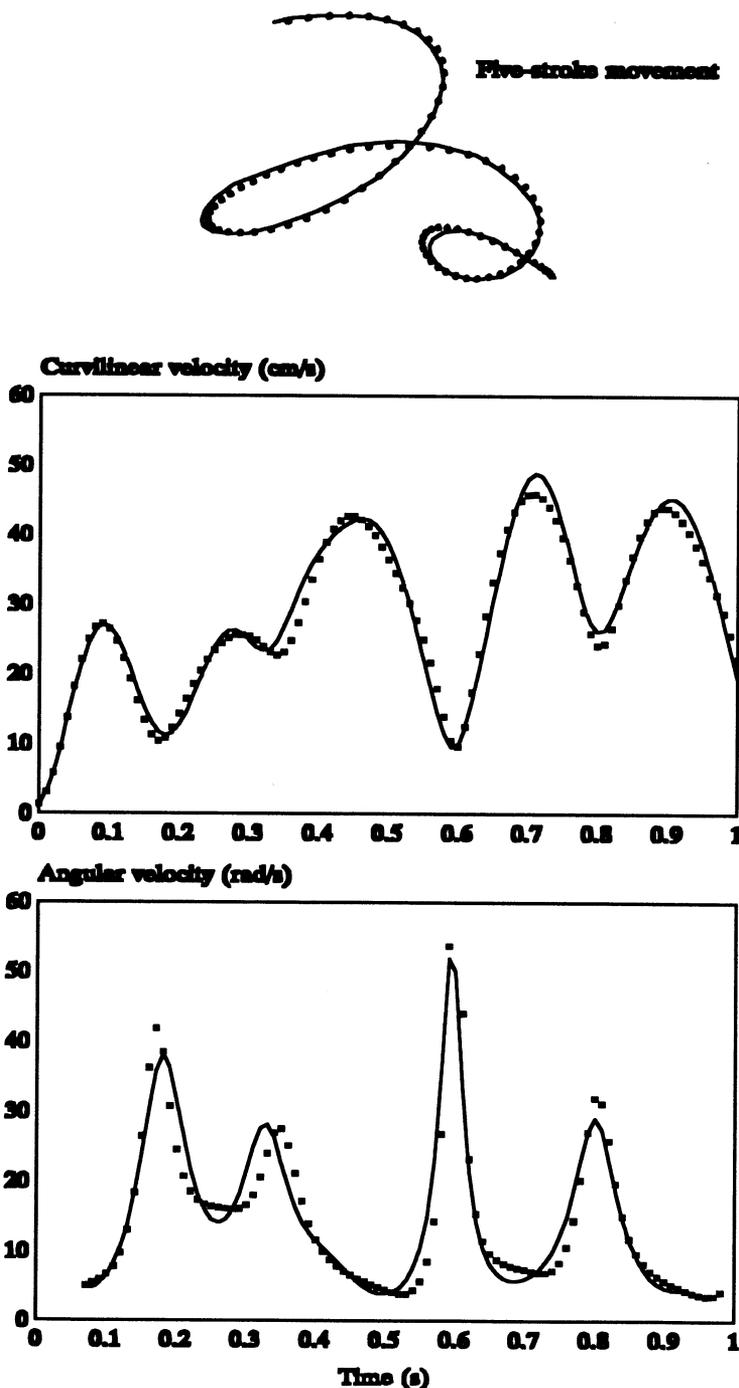


Fig. 6. Five-stroke movement, original (solid lines) and generated after extraction (squares).

dently of the resulting trajectory, and the resulting movement will reach the same final point with the desired spatial precision whatever the starting times of the individual movements are. This property of vectorial algebra enables us to say that the activation time of single strokes (in complex movements) will have an influence on the length and shape of the resulting path, but will not necessarily affect the final target.

This important property of the vectorial delta-log-

normal model can explain some phenomena reported in psychophysics, where human behaviour in the control of perturbed planar movements was studied. A typical experiment requires asking a subject to make a simple movement from a point *A* to a target point *B*, and, while the movement is being executed, changing the target *B* to a point *C*. Goodale et al. (1986), Pelisson et al. (1986) and Martin and Prablanc (1992) have reported that in such an experiment the subject does not

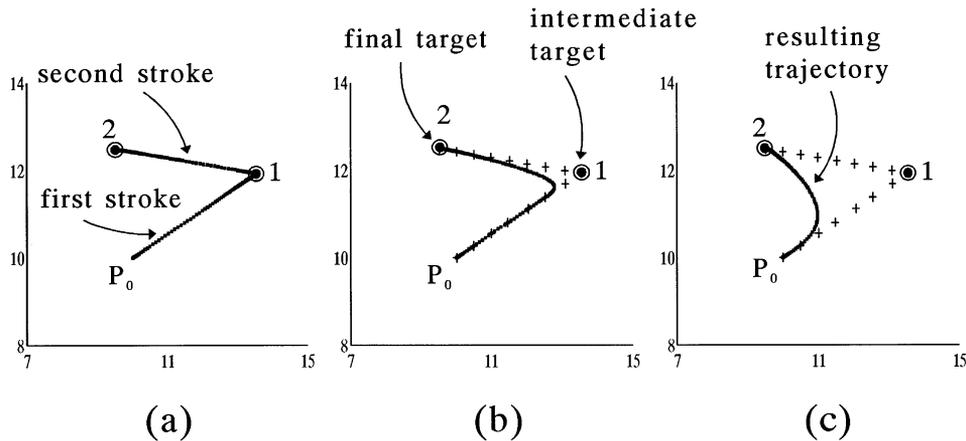


Fig. 7. Effect of stroke overlap on the resulting movement: (a) no overlap; (b) moderate overlap; (c) significant overlap.

use any visual retroaction to see the position of his hand, to correct the trajectory already being executed. Pelisson et al. (1986) reported some unconscious process that enables the subject to correct the hand trajectory to the final target. What we claim here with the vectorial delta-lognormal model is that, in fact, there is no need for the subject to know where the hand is, when he or she starts the second movement. In this case, the subject only needs to evaluate the distance between the previous and the new target and the spatial precision needed to reach this new target. Once the subject has evaluated the distance between the two targets and the spatial precision needed, the new set of commands is specified and it is independent of the position of the hand at that time. The vectorial properties of movement generation are sufficient to explain why the adjustment of the trajectory is independent of the vision and the instantaneous position of the hand. This idea of not aborting the first movement and generating a second independent movement vectorially added to the first to correct the trajectory in perturbed movements is also supported by Flash and Henis (1991). What we argue here is that all the movements in this case obey the delta-lognormal law (Eqs. (1) and (2)). Experiments will have to be conducted to confirm this hypothesis, using the method described in this paper.

Fig. 8 also suggests that the vectorial delta-lognormal model can be used to recover and analyse a representation of an action plan from a given signal. Indeed, the model assumes that, at some level of representation, a movement is represented with a topology-preserving map as a sequence of strokes to virtual targets. This map can be simulated, for example, using a grid of leaky integrators (Privitera and Plamondon, 1995). Taking the virtual targets as input, the global activation of the map, as described by competitive population coding, will be strictly correlated with the kinematic state of the ongoing external movement. In this context,

synchronisation instants between consecutive motor strokes can be detected; that is, the temporal synchronisation of a stroke is partly determined by the shape itself. We have shown, among other things, that such a model could be used to control both the generation and the learning of target-directed movements (Plamondon and Privitera, 1996).

6. Conclusion

In this paper, a new method based on the kinematic theory of rapid human movement has been proposed to represent, generate and analyse simple and complex planar movements. The vectorial delta-lognormal model involved provides a representation of an action plan that can be used to reproduce movements in both the kinematic and the static domains. To represent a basic movement (a single stroke), the model requires a set of nine parameters (plus the starting point of the first stroke) that totally describes the movement in both domains. Complex movements are described by the model as a vectorial summation of time-overlapped single strokes. It is shown that the relationship and the shape of curvilinear and angular velocities are the resultant of the vectorial summation of single vectors, and can be very well approximated by the model.

A method for parameter estimation and optimisation is proposed in this paper. A two-step approach is proposed, one step to approximate the parameters and a second to optimise the solution by means of a robust technique for nonlinear regression analysis. The optimality of the solution of a nonlinear regression technique applied to a multi-domain problem is not guaranteed, but a realistic solution can be obtained for the movement representation problem, kinematic analysis and movement control. Even if the solution found is not necessarily either unique or optimal, one might assume that the repetition of the same estimation

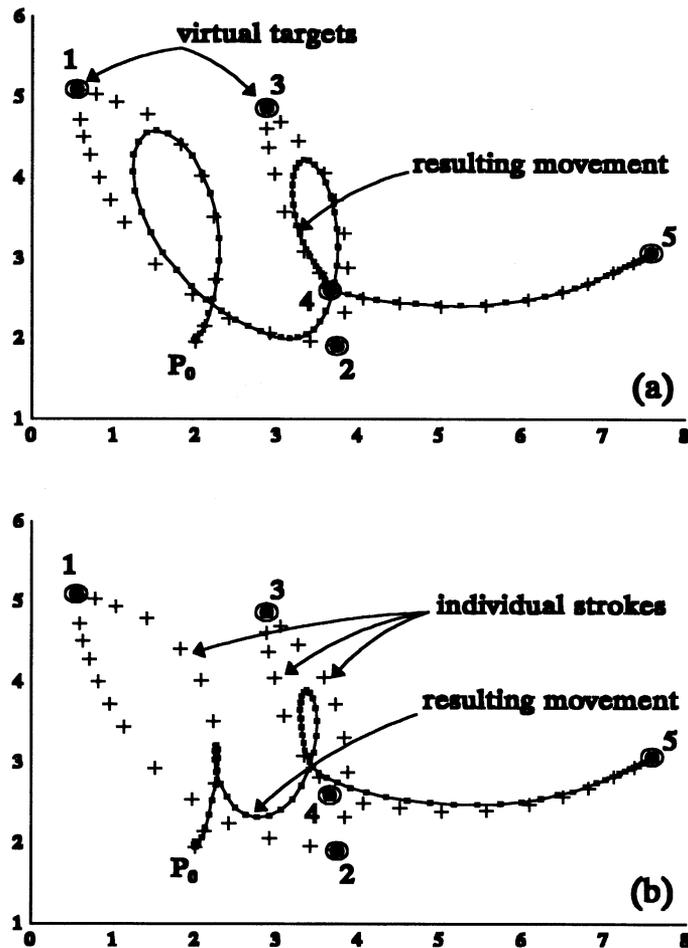


Fig. 8. Effect of starting time on complex movements: (a) moderate stroke overlap; (b) significant stroke overlap.

method over a large set of individual data will lead to a set of parameter classes that describes typical experimental conditions. Intra-class and inter-class statistics will then be useful to provide a new method for analysing and characterising changes that occur between these experimental conditions.

Parameter extraction can be used in various applications and domains, especially in neuroscience, to study simple movement representation and control, oscillatory movement analysis and complex movement planning and representation (Plamondon, 1998). Experiments on perturbed movements, as shown briefly in this paper, can also be explained by the model and new ways to analyse and understand the effects of partially hidden movements, trajectory correction strategies and retroaction can also be explored.

All these analyses can be conducted because the delta-lognormal model theoretically provides some cues to distinguish between single ballistic strokes (with up to three velocity peaks) and complex disturbed movements that also encompass multiple velocity peaks (see Fig. 4). Indeed, a principle of stroke minimisation can be assumed that will favour the reconstruction of a

signal using the minimum number of strokes. Using this approach, it is found that optimal reconstruction of multi-stroke movements is generally characterised by the fact that successive strokes are generated in different (and mostly nearly opposite) directions. If an analysis-by-synthesis leads to successive movements with almost the same direction, a multi-peak stroke can be assumed to be hidden in the signal, instead of a sequence of two or three single-peak strokes.

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