

THE LAW RELATING THE KINEMATIC AND FIGURAL ASPECTS OF DRAWING MOVEMENTS *

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Considerable evidence exists that the velocity of execution of handwriting and drawing movements depends on some global metric properties of the movement (size, linear extent etc.). Recent experiments have demonstrated that the instantaneous velocity also depends on the local curvature of the trajectory, that is, on the differential geometrical properties of the movement. In this paper we investigate further the role of the differential factors. Experiments are described in which drawing movements of simple geometrical forms and scribbles are performed either freely and extemporaneously, or in the presence of external constraints. It is shown that, at any time during the movement, the velocity component related to differential factors only depends on the value of the curvature of the trajectory at the same time (no dynamics). The relation can be described quantitatively as a specific Power Law and applies to all movements considered here, including those which are performed by following the edge of a template. The fact that the velocity of execution increases with the radius of curvature implies a built-in tendency of the motor control system to keep angular velocity relatively constant and qualifies the Isogony Principle proposed previously. The specific exponent of the Power Law suggests a possible interpretation of this empirical relation.

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Introduction

The organization of complex movements involves several constraints, internal to the Central Nervous System, which take the form of Invariance Principles (Bernstein 1967; Greene 1972; Schmidt 1982; Stelmach and Requin 1980). Some of these principles relate form and kinematics of the movement. In particular, the study of handwriting and drawing has revealed a systematic relationship between the instantaneous tangential velocity of the pen's tip and the geometrical properties of the trajectory (Viviani and McCollum 1983; Viviani and Terzuolo 1980, 1982). Behaviorally, this relationship results in the so-called Isogony Principle (Viviani and McCollum 1983; Viviani and Terzuolo 1982) which states that the *angular velocity* of these movements tends to remain constant when the radius of curvature changes. For some particularly simple trajectories, the principle applies strictly throughout

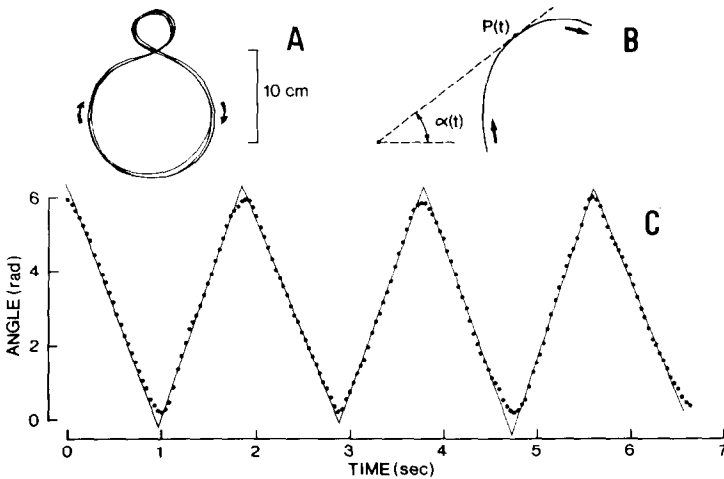


Fig. 1. *Isogony*. The results illustrate the Isogony Principle in a simple closed pattern (A). The principle states that, in drawing movements, equal angles are described in equal times. In panel C is shown the time course of the angle $\alpha(t)$ that the tangent to the trajectory forms with an arbitrary reference (see panel B). A linear interpolation (continuous lines) fits quite accurately both the increasing and decreasing portions of the graph corresponding to the larger and smaller loop of the pattern, respectively. The average angular velocity in the two loops (slope of the corresponding graphs) differs by less than 20%. Thus, the time of execution of the loops tends to be independent of their size. Note that the example illustrated is somewhat extreme. In general, the extent to which the angular velocity is independent of the radius of curvature is a function of the coupling between the execution of the two parts of the pattern (see Lacquaniti et al. (1983) for a full discussion of this point).

the entire movement (see fig. 1). In general, though, the trajectory appears to be composed of successive, individually planned segments, each characterized by a distinct value of the angular velocity. These rather abrupt changes in the angular velocity take place every time that the trajectory presents a well-defined geometrical singularity (points of inflection, cuspids), but can also occur away from these points (Viviani and Terzuolo 1982).

Recent studies (Lacquaniti et al. 1983; Viviani and McCollum 1983) have suggested that the instantaneous tangential velocity can be factored out into at least two components, one of which depends on the local differential properties of the trajectory, and the other on some more global aspects of it. More specifically, it appears that the instantaneous velocity $V(t)$ within each segment can be expressed as the product of a variable term which is a function of the radius of curvature $R(t)$ of the trajectory and of a gain factor k which remains constant during the execution of the segment:

$$V(t) = kg(R(t))$$

The gain factor is a monotonically increasing function of the length of the segment to be executed. Moreover, it can also depend on the total linear extent of the movement in a complex and, as yet, poorly understood way. Apparently, it reflects the well-known tendency in voluntary movements to maintain the execution time approximately independent of the length of the trajectory (Fitts 1954; Viviani and Terzuolo 1982). A more detailed analysis of the gain factor and a discussion of its possible significance vis à vis the notion of motor preparation is presented in a separate paper (Lacquaniti et al. 1983).

The present paper focusses, instead, upon the exact nature of the relation $g(R)$ between the tangential velocity and the radius of curvature. We shall demonstrate that one specific law applies to a large variety of trajectories and that such law is valid both for learned movements and movements produced extemporaneously. Moreover, we will also show that the same relation applies in the presence of external constraints to the movement, including mechanical ones.

Methods

The x- and y-coordinates of the drawing movement at the pen's tip were recorded with two different instruments. In some experiments an ultrasonic device was used (Graph Pen, accuracy: 0.1 mm; sampling rate: 100 Hz); in other experiments an electromagnetic digitizing table (Calcomp 622 RP, accuracy: 0.025 mm, sampling rate: 100 Hz). In both cases the writing implement closely resembles an ordinary ballpen and the set-up permitted a comfortable writing posture.

Time derivatives of the displacement data were calculated with the Lagrange 5-points formula after smoothing the raw data with a (double-sided exponential) numerical, low-pass filter (cut-off frequency 50 Hz).

In some experiments the movements were performed by following with the pen's tip the inner edge of a template. For ellipses and pseudo-ellipses (see Results section), the templates were cut into Plexiglas by a computer-controlled machine and were extremely accurate and smooth. The random form of fig. 2 was, instead, cut manually and was less smooth.

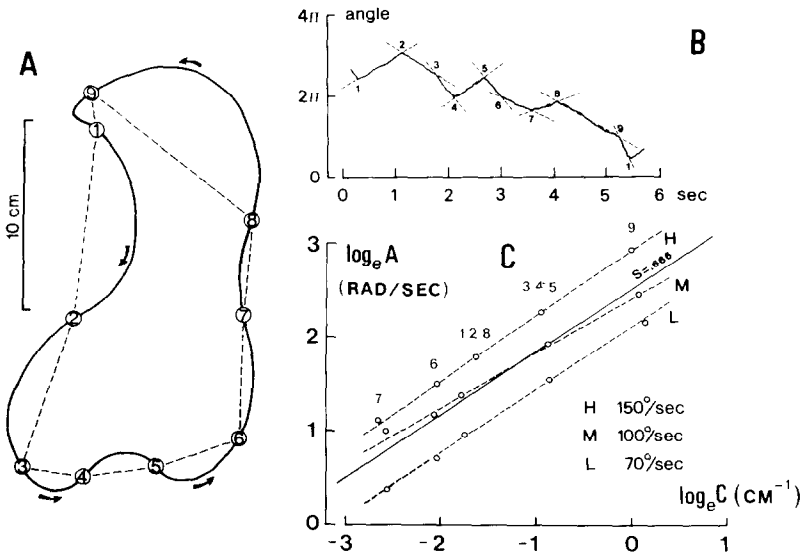


Fig. 2. Relation between absolute angular velocity and curvature for constrained movements. The template traced in this experiment (panel A) is composed of a random succession of arcs. In B the time course of the angle $\alpha(t)$ (cf. fig. 1B) for one complete cycle of the movement. The graph can be approximated by the indicated sequence of rectilinear segments. Numbers permit to identify the points on the trajectory where the average angular velocity changes. Panel C shows the relation between average angular velocity A (radian/sec) and average curvature C (absolute values, cm^{-1}) over the different portions of the trajectory (see text), for three speeds of execution (High, Medium, Low). The data points summarize the results in three subjects. Power functions accurately fit the data points at all speed as indicated by the dashed lines. A theoretical power two-third function (continuous line) is plotted for reference.

In other experiments the movements were unconstrained. However, in cases which will be specified under Results, the desired trajectory was printed on the working plane to provide the *S* with a general guidance. When closed trajectories were drawn, *S*s traced the same pattern over and over without interruption. Scribbles were produced continuously and extemporaneously. After a few seconds for warming up, and without warning, the computer recorded a 10 sec sample of the movement. In the case of the elliptic spiral the *S* decided when to initiate the movement and the recording started immediately afterward.

Four male adults volunteered for the experiments.

Results

The first task will be to provide a quantitative description of the relation $V(t) = kg(R(t))$. Since the radius of curvature R is infinite at the inflection points, it is actually easier to investigate the equivalent relation between angular velocity ($A(t) = V(t)/R(t)$) and curvature ($C(t) = 1/R(t)$) which can be formally written as: $A(t) = kf(C(t))$.

Let us first consider the relation between the *average* values of A and C over non-overlapping segments of the movement. The basic experiment consists in tracing a curvilinear trajectory with the help of a template. As seen in fig. 2A, the trajectory consists of a random succession of arcs with mildly varying curvatures. In each trial the *S*s were asked to go around the inner rim of the template continuously for 10 sec, with a tempo suggested by a metronome. Three values of the tempo were considered corresponding to the following speeds of rotation: High = 150°/sec, Medium = 100°/sec, Low = 70°/sec. Three *S*s participated in the experiment.

Since the trajectory is imposed, the motion is fully described by a single dynamic parameter, namely the angle $\alpha = \alpha(t)$ between the tangent to the trajectory at the tip of the pen and an arbitrarily fixed reference. A typical record for a complete loop around the template is shown in fig. 2B. It is obvious that $\alpha(t)$ can be adequately approximated by a series of contiguous linear segments (dashed lines) with different slopes. The endpoints of each segment are labelled with numbers which permit the identification of the corresponding points along the trajectory. Note that the changes in slope mostly occur at points of inflection (points 1, 2, 4, 5, 7, 8) and that, as expected, the longest segments are those with the largest radius of curvature.

The average angular velocity for each segment was estimated by the slope of the corresponding linear interpolation of the $\alpha = \alpha(t)$ plot (see fig. 2B). For all *S*s and all segments the relative variability across trials of the angular velocity (σ/m) was of the order of 10%. Moreover, the performance of the three *S*s was remarkably similar as shown by the data of table 1. Thus, the angular velocity for each segment was averaged over all loops of a trial, all trials (at least 5) and all *S*s. Since the absolute values of A and C for the segments 1-2, 2-3, 8-9 and 3-4, 4-5, 5-6 were not appreciably different, they were lumped together, respectively. Panel C of fig. 2 summarizes the relation between A and C for each value of the tempo separately.

The data demonstrate that the relation between A and C can be described ade-

Table 1
Normalized angular velocities. For three subjects (S_1 , S_2 , S_3) are indicated the averages over the three speeds of execution of the angular velocity in each segment normalized to the mean angular velocity in one complete movement cycle. The relative changes in angular velocity from segment to segment are very similar in all subjects.

| | Segments | | | | | | | | |
|-------|----------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| S_1 | 0.595 | 0.731 | 1.189 | 1.073 | 1.246 | 0.548 | 0.401 | 0.713 | 2.490 |
| S_2 | 0.733 | 0.755 | 1.327 | 1.027 | 1.244 | 0.550 | 0.545 | 0.644 | 2.169 |
| S_3 | 0.601 | 0.847 | 1.525 | 1.028 | 1.117 | 0.536 | 0.428 | 0.703 | 2.215 |
| Av. | 0.643 | 0.778 | 1.347 | 1.043 | 1.202 | 0.545 | 0.458 | 0.687 | 2.291 |

quately, at all speeds, by a linear function (dashed lines) fitted to the data points. By comparison with the solid reference line, it is seen that the slopes of the dashed lines are close to $\frac{2}{3}$.

Next we ask the following question: does the relation between curvature and angular velocity depend on the frequency spectrum of these quantities as in most dynamical systems (viz. integrators, differentiators)? This question can only be tackled by considering the instantaneous values, ($C(t)$) and ($A(t)$), of these quantities. Specifically, if we assume that changes in angular velocity are causally dependent upon corresponding changes in curvature, and considering C and A as the input and the output of a dynamical system, we can address the stated question with a procedure equivalent to the step-response paradigm utilized in system analysis. The simplest closed form which contains step changes in curvature is obtained by joining two pairs of circular arcs with different radii, R_1 and R_2 . The length of the arcs for each value of R_1 and R_2 can be so chosen as to get a smooth contact at their junctions. Thus, despite the presence of jump discontinuities in curvature, the tangent to the form does not contain first order singularities. A set of 11 such forms were manufactured in which the radii R_1 and R_2 vary from 5 to 10 cm and from 15 to 10 cm, respectively, both in steps of 0.5 cm (in all cases $R_1 + R_2 = 20$ cm). Thus, the shape of the templates varies

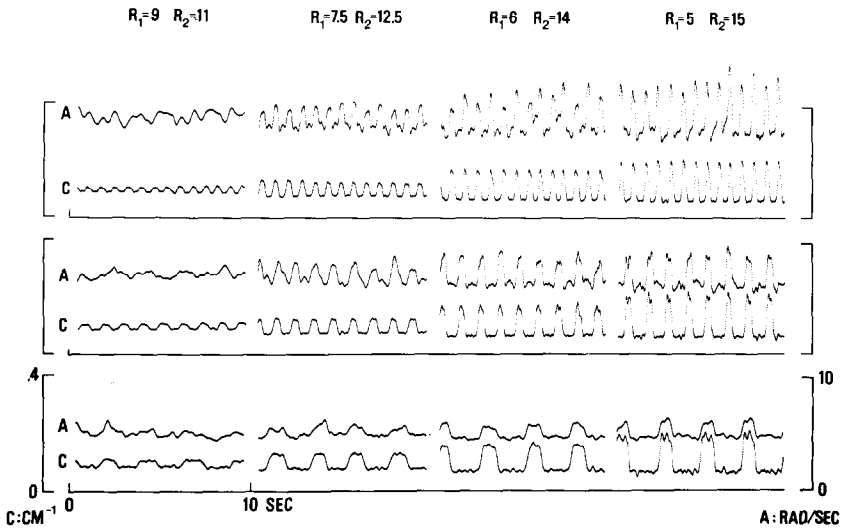


Fig. 3. Step-response analysis of the relation between curvature (C) and angular velocity (A). Templates were used in which two pairs of circular arcs with radius R_1 and R_2 were joined with smooth contact. The templates have continuous contours but contain step changes in curvature. Instantaneous values of A and C are plotted for several cycles of movement, at three different speeds of execution (decreasing from top to bottom), and four representative values of R_1 and R_2 . Changes in angular velocity only reflect the instantaneous values of the curvature with no appreciable dynamics. When the step changes in curvature are small, the angular velocity contains components other than those present in the input.

progressively from a circle to a very elongated pseudo-ellipsis, while the perimeter is kept constant. The experiments were performed by two *Ss* with the same modalities as in the previous one.

Fig. 3 shows representative examples of the instantaneous value $C(t)$ and $A(t)$ for the indicated values of R_1 and R_2 and for three speeds of execution. The data demonstrate that (i) the angular velocity changes along the perimeter *pari passu* with the radius of curvature; (ii) when the form of the template approaches a circle, as in the three leftmost panels, the angular velocity contains harmonic components other than those present in the input $C(t)$ (these components become less and less appreciable as the difference between radii increases); and (iii) there is no measurable time delay between the corresponding wave-forms of C and A . Since any frequency-dependent dynamical system that is physically realizable introduces such phase shifts, we must conclude that the $A-C$ relationship does not depend on the frequency spectrum of these variables.

Furthermore, the results of this experiment validate the two-third power law already demonstrated in fig. 2 for complex trajectories. This is shown in fig. 4 where the data

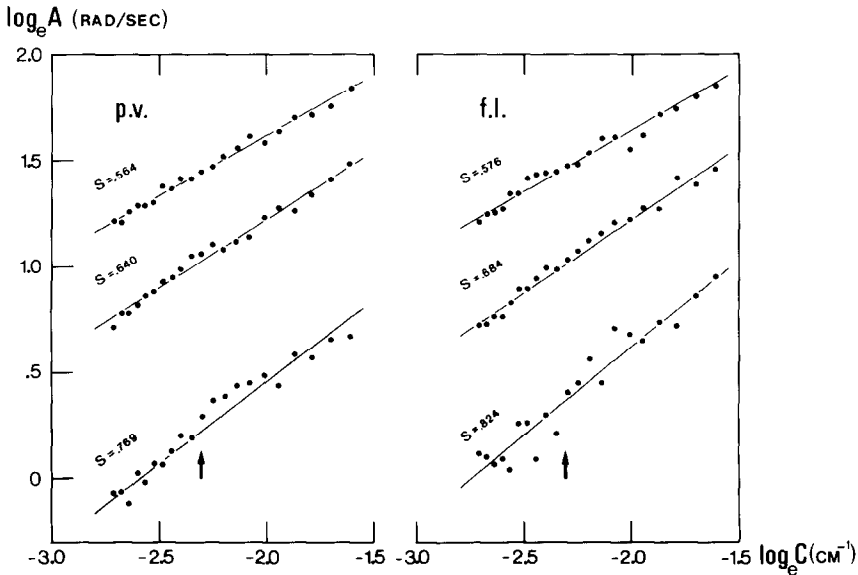


Fig. 4. The two-third power law obtained in the step-response experiment. Data from two subjects (p.v. and f.l.) under three speeds of execution. (One set of 21 data points for each speed. From top to bottom the average speeds are 245, 165 and 80 degrees/sec.) For a given speed each template produces a pair of data points in the corresponding $A-C$ plot, one for the more curved and the other for the less curved portions of the trajectory. These two points are placed symmetrically with respect to the (unpaired) central point in each plot (arrows) which corresponds to the circular template ($R_1 = R_2$). The data points are fit by power functions. The exponents of the functions are indicated by the slopes (S) of the interpolating lines. Notice that the exponent decreases with speed increase.

for the two S s are plotted separately. The results for each template and each speed are summarized by a pair of data points, one being the average of C and A over the arc with radius R_1 and the other over the arc with radius R_2 . The slopes of the $C-A$

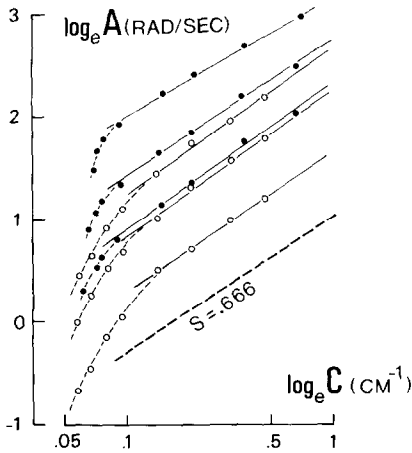


Fig. 5. The $C-A$ relationship for four elliptic templates with different eccentricities. Two sets of data are shown: the filled points correspond to the data obtained with one subject using the Calcomp Digitizing Table; the empty circles correspond to averages from three subjects using the Graph Pen System (see Methods). As in fig. 4, each template yields two points, one per pair of segments with quasi-constant angular velocity (see text). Note that the relationship between A and C in a log-log scale is again linear, except for the points at the lower end of the curvature range.

relation for each speed, given in the figure above the corresponding linear interpolation, are in good agreement with the conclusion already reached in the case of the complex form.

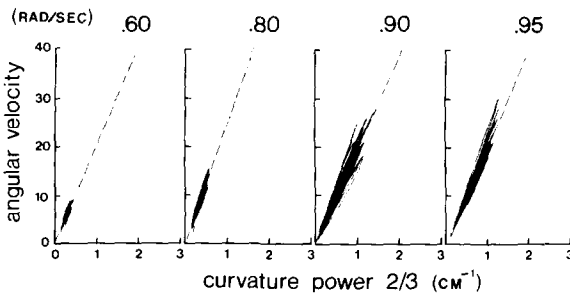


Fig. 6. The two-third power law also applies to the instantaneous values. Graphs describe the instantaneous relation between angular velocity and the $\frac{2}{3}$ power of the curvature for the free hand drawing of ellipses. For each of the indicated value of the eccentricity, the relation takes the form of a narrow bundle of linear segments. The average slope of the bundles appears to be independent of the eccentricity.

A further generalization of the power law was obtained by utilizing true elliptic templates in which the radius of curvature varies continuously. Fig. 5 reports the results of such an experiment in which four ellipses with different eccentricities (0.60, 0.80, 0.90, 0.95) were used. Four Ss participated in this experiment.

Previous work (Viviani and Terzuolo 1982) has shown that in the case of ellipses the angular velocity oscillates between two approximately constant and distinct values, a higher one where the curvature is higher (at the poles) and a lower one where the curvature

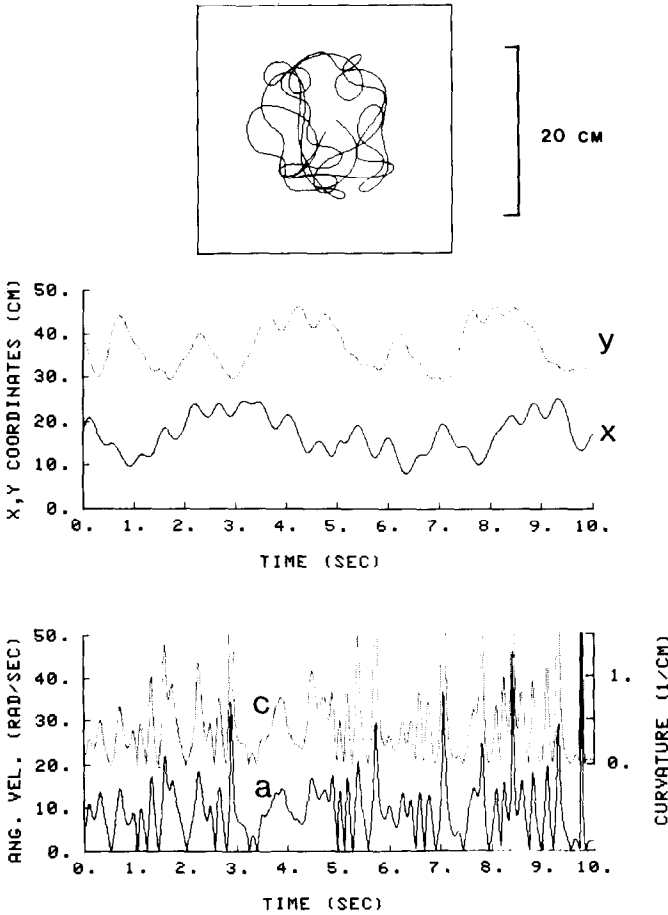


Fig. 7. Free-hand drawing of an extemporaneous movement. In A, a representative example of scribble. In B, the time course of x (thick line) and y (thin line) coordinates of the movement: no obvious relationship exists between the two variables. In C, the time course of the angular velocity (thick line labelled a) and that of the curvature (thin line labelled c). The remarkable similarity between the two curves demonstrates also in the case of free and erratic movements the direct relationship between the instantaneous values of angular velocity and curvature.

is lower. The transitions between these values suggests a division of the movement in two pairs of symmetrical segments. It is therefore justifiable to average both curvature and angular velocity over each pair of segments yielding two data points for each template as in the previous case. In the range of curvature values covered in our

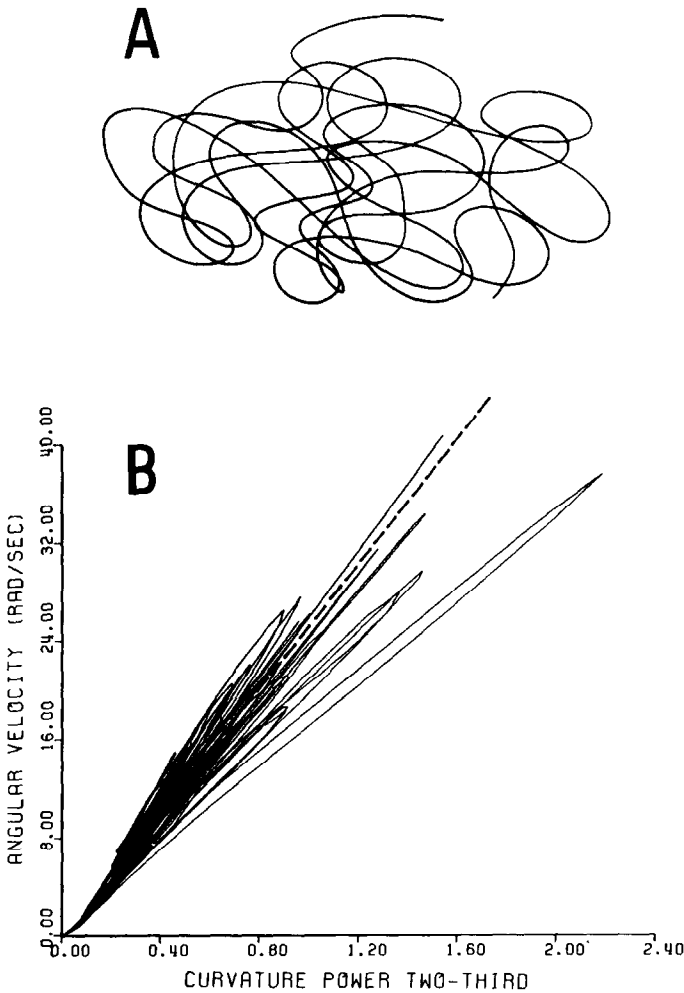


Fig. 8. The two-third power law holds true for scribbles. In A, a representative example of scribble. In B, the instantaneous values of the angular velocity are plotted against the power two-third of the curvature (cf. fig. 6). The relation between the two variables is piece-wise linear. Each segment in the bundle, corresponding to non-overlapping segments of the trajectory, has a different slope. The clustering of slopes around the average (dashed line) is representative of the results for this type of experiment.

experiment the slope of the interpolating lines is again close to $\frac{2}{3}$ for the data points which essentially describe the C - A relation in the polar regions of the ellipses. The points corresponding to the flatter regions (connected by the dashed curves) depart, however, from the power law. One possible explanation for this departure is the methodological difficulty to determine the transition between segments, and the fact that small differences in determining this transition result in large differences in the averages for the flatter regions, without affecting the other data points.

The power law verified so far at the level of the average values of C and A , also applies to the instantaneous values of these quantities. A simple demonstration of this fact is provided by the data obtained with the free hand drawing of ellipses of different eccentricity. For this experiment an outline of the four elliptical templates used previously was printed to provide a guidance for the otherwise unconstrained movements. Fig. 6 reports data from one S . Similar results were obtained in three other S s. Each panel in the figure shows, for the indicated eccentricity, the instantaneous angular velocity plotted against the two-third power of the curvature. The validity of the power law in this case is demonstrated by the very high correlation between the two variables. The average slope of the relation indicated by the linear regression (broken lines) appears to be independent of the eccentricity.

The strongest confirmation of the general validity of the power law is finally provided by removing all constraints from the movement, as in the case of scribbles

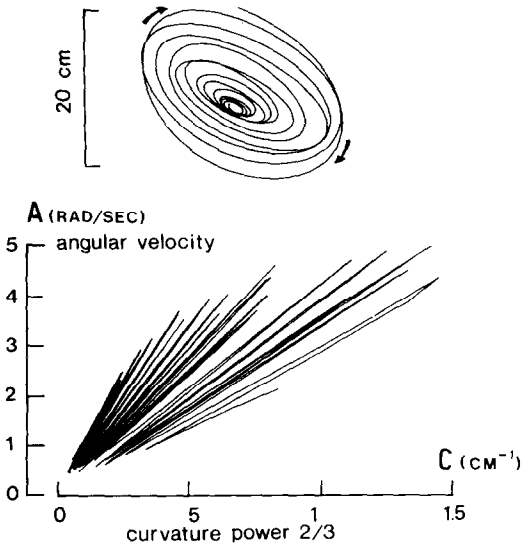


Fig. 9. *The effect of size on the two-third power law.* Relationship between angular velocity and curvature in the case of an elliptic spiral. The two-third power law applies individually to each loop of the spiral and gives rise to a regular fanning of segments in the C - A plot. As the movement winds up toward the center, the slope of the segments decreases. Thus, size only affects the proportionality constant of the power law.

drawn extemporaneously. Fig. 7 illustrates a representative example in one *S*: in A the actual scribble; in B the temporal evolution of the x-y-coordinates; in C the corresponding evolution of the angular velocity and curvature. The main point of the figure is that while no simple relationship can be seen between x- and y-coordinates, the angular velocity very closely duplicates the evolution of the curvature, as already shown in the much simpler case of the pseudo-ellipses (fig. 3). The quantitative aspects of the C-A relationship are demonstrated in fig. 8 with the same type of representation as in fig. 6. Each part of the trajectory gives rise to a linear segment which obeys the two-third power law. The slope of each segment is somewhat variable, and should be identified with the constant k in equation $A(t) = kf(C(t))$. This subject will be taken up in a separate paper (Lacquaniti et al. 1983).

Here we shall simply provide the experimental evidence that the constant k depends on the metric properties of the trajectory. The data in the diagram of fig. 9 pertain to the free-hand drawing of the elliptic spiral shown in the upper part of the figure. The random scatter of slopes typically observed in the case of scribbles now becomes a regular fanning of individual linear segments.

Each elliptic loop has two points of maximum curvature and two points of minimum curvature. Therefore, the execution of the loop is described by four segments, representing the successive transitions between these extreme values. The steepest segments correspond to the innermost loops. Thus, a regular monotonically increasing function relates the slope of the segments, that is the constant k , to the size of the loop. This finding confirms the validity of the multiplicative decomposition described elsewhere in the case of simple trajectories with piece-wise constant curvature (Viviani and McCollum 1983).

Discussion

All the results presented confirm the existence of a specific law relating the instantaneous tangential (and angular) velocity of drawing movements to the radius of curvature of the trajectory. The law, which takes the form of a power function, applies to a large variety of trajectories, ranging from simple closed patterns to extemporaneously drawn scribbles. Furthermore, the law is robust in that it remains valid even when the movement is mechanically constrained, or the tempo of execution is imposed.

As a first point of the discussion we will consider the relation between the two-third power law and the Isogony Principle mentioned in the Introduction. When the principle applies strictly – as in the example of fig. 1 – it implies a proportional relationship between the instantaneous tangential velocity $V(t)$ and the radius of curvature $R(t)$. On the other hand, the two-third power law – which can also be written

as $V(t) = kR(t)^{\frac{1}{3}}$ – implies a less than linear relationship. Thus, the dependence of the instantaneous velocity from the local differential properties of the trajectory only accounts for part of the tendency to keep the angular velocity constant. An additional contribution in the same direction must necessarily come from the gain factor k which appears in the proposed formulation of the two-third power law. To suggest a possible way in which the gain factor may indeed provide the required contribution, let us consider again the Figure Eight pattern shown in fig. 1. If we assume – as detailed elsewhere (Lacquaniti et al. 1983; Viviani and McCollum 1983) – that the gain factor in each of the two loops which compose the pattern is a power function of the perimeter, we get:

$$V_1(t) = k'(2\pi R_1(t))^b R_1(t)^{\frac{1}{3}} = (2\pi)^b k' R_1(t)^{b+\frac{1}{3}}$$

$$V_2(t) = k'(2\pi R_2(t))^b R_2(t)^{\frac{1}{3}} = (2\pi)^b k' R_2(t)^{b+\frac{1}{3}}$$

in the two loops respectively (k' is a constant which depends on the tempo of the movement). Since the value of the exponent b is of the order of 0.55 (Lacquaniti et al. 1983), and therefore $b + \frac{1}{3} \approx 1$, it follows that $V_1/R_1 \approx V_2/R_2$. Thus, the observed constancy of the angular velocity may be explained by the joint action of the two-third power law and of the gain factor. A similar argument may be used to generalize the above conclusion to any pattern composed of segments which can be approximated by circular arcs. Ultimately, we surmise that whenever the length of several pattern segments is approximately proportional to their respective average radii of curvature, the angular velocity for these segments is roughly the same. It should be stressed, however, that in some cases the relation between the gain factor and the length of the segments which compose a pattern may be vastly more complex than the simple one described for the Figure Eight pattern. For instance, strict Isogony is certainly not attained in the execution of the pseudo-ellipses (see fig. 3).

The second point of the discussion is the significance of the two-third power law from the point of view of the control of movement. To begin with, let us note that the very existence of a precise relation between the instantaneous velocity and the local differential properties of the trajectory qualifies the original proposition that form determines movement

kinematics (Viviani and Terzuolo 1982). It can in fact be shown (Viviani and Terzuolo 1983) that, given the validity of the two-third power law, it is possible to predict quantitatively the kinematics of the movement on the sole basis of the equations of the trajectory. As a working hypothesis, we can then postulate the existence of control processes that actually translate an essentially static description of the form of the movement into dynamic commands compatible with the execution of that form. If so, the planning and the preparation of the movement would be mostly concerned with the specification of the intended trajectory and of the overall tempo, while the exact time course of velocity and acceleration would be specified by the postulated control processes.

Finally, we will consider the question: Should any special significance be attached to the value of the exponent, or is the value $\frac{2}{3}$ purely coincidental? At this stage we can only provide a tentative answer based on the following reasoning. Some authors (Denier van der Gon and Thuring 1965; Hollerbach 1982) have postulated that the formation of the trajectory in drawing and handwriting movements is achieved by composing two harmonic oscillations with orthogonal directions (Mass-Spring theory). Thus, the basic form of the parametric law of motion would be:

$$\dot{x}(t) = A \sin \omega t$$

$$\dot{y}(t) = B \sin(\omega t + \varphi)$$

where it is assumed that amplitude and phase of the oscillations are modulated by efferent motor commands. In this case, straightforward calculations show that the two-third power law becomes a mathematical identity. However, the converse is obviously not true: the fact that a specific movement satisfies the two-third power law does not imply that its law of motion has the general parametric form given above (Viviani and Terzuolo 1983). Indeed, the law applies to cases where the Mass-Spring theory is clearly inadequate to describe the law of motion (cf. fig. 7B). In conclusion, while in many cases the analysis of actual drawing movements is at variance with the predictions of the Mass-Spring theory, the processes that control the movements – whatever their logic – result into a global behavior which exhibit one abstract relational feature of the harmonic model. According to this line of

reasoning we must conclude that the specific value of power law exponent reflects an important aspect of the control logic.

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