ELSEVIER

Human Movement Science 18 (1999) 485–524

HUMAN MOVEMENT SCIENCE

www.elsevier.com/locate/humov

A nonlinear analysis of the temporal characteristics of handwriting

M.G. Longstaff *, R.A. Heath [1]

Department of Psychology, University of Newcastle, University Drive, Callaghan, NSW 2308, Australia

Abstract

Motor skills provide us with an almost infinite variety of ways in which we can interact with the world. This paper considers the problem of how the psychomotor system translates a stable motor memory into an invariant spatial output within an infinitely variable biomechanical and environmental context. Initially the validity of a novel methodology, based on the concatenation of handwriting velocity data over several trials to form long time series, combined with singular value decomposition to reduce noise, was confirmed. The data analyzed were the horizontal and vertical velocity of the stylus as eight participants wrote the pseudo-word *madronal* on a computer graphics tablet. Nonlinear dynamic analysis techniques such as examination of delay portraits, as well as calculation of the correlation dimension and Lyapunov spectra were applied to test the hypothesis that handwriting velocity profiles are chaotic. The findings that the largest Lyapunov exponents were positive, the sums of Lyapunov spectra components were negative and the correlation dimensions were low and fractional supported this hypothesis. We conclude by proposing that the psychomotor actions found in handwriting are a product of a chaotic dynamic process whose initial conditions depend on the environmental and biomechanical context. © 1999 Elsevier Science B.V. All rights reserved.

PsycINFO classification: 2330

* Corresponding author. Tel.: +61-249215958; fax: +61-249216980; e-mail: mitchell@hiplab.newcastle.edu.au

[1] E-mail: rheath@hiplab.newcastle.edu.au

PII: S 0 1 6 7 - 9 4 5 7 (9 9) 0 0 0 2 8 - 7

Keywords: Coordinative structures; Degrees of freedom problem; Nonlinear dynamic analysis; Rhythmic movement; Motor memory; Chaos theory

1. Introduction

Handwriting is a typical dynamic motor skill that requires the integration of cognitive and biomechanical systems to produce an output that is stable and reproducible. When we intend to write the letter ***a***, the goal is that others will recognize it as such. Once this skill is learnt it can be performed automatically, with little need for active conscious control. Psychomotor skills have been found to be controlled by quite complex dynamical systems whose properties and structures are yet to be fully described and understood.

A key question within this field is called "The degrees of freedom problem". The degrees of freedom of any system are equal to the minimum number of variables needed to fully describe the state of that system. However, there are many more degrees of freedom available to the cognitive/biomechanical system than are needed to produce a given output. Bernstein (1967) discusses the idea that the coordination of motor movements involves a reduction in the number of degrees of freedom. This is partly achieved by combining functional groups of muscles that are constrained to act as a single unit (Kay, 1988). These 'functional synergies' are controlled by the dynamical system with subsequent movements being modified using feedback if time permits (Sheridan, 1984).

The problem is how does the psychomotor system translate the stable motor memory into an invariant spatial output within an infinitely variable biomechanical and environmental context? We need to discover how the psychomotor system picks and controls a particular set of degrees of freedom to produce the letter ***a*** in a given environment, when it has an infinite number of degrees of freedom from which to choose from. A further goal of this research is to ascertain how this information is represented in memory.

What is needed to solve this problem is a motor memory comprised of the boundary conditions of a functionally specific coordinate structure (Kay, 1988; Mitra, Amazeen & Turvey, 1998; Mitra, Riley & Turvey, 1997) which can generate infinite complexity. Recently there has been a great deal of interest in studying dynamical systems and whether it is possible to model real world phenomena in terms of dynamic equations. Of particular interest are those equations that display what is known as chaotic behavior. Chaotic processes are typically generated by simple low dimensional dynamical systems that are sensitively dependent on initial conditions. Slightly different initial values can lead to an infinite number of vastly different system outputs. These processes can produce an infinite range of characteristic outputs.

If handwriting was found to be chaotic then such a finding might solve the degrees of freedom problem. Output that is of the order of complexity found in skilled movement within a rapidly changing cognitive, environmental and biomechanical context could be produced by the variation of a small number of important parameters. As a result the seemingly infinite degrees of freedom would then collapse to the few degrees of freedom actually observed during movement.

2. Chaos in handwriting

Findings from several fields of research suggest that handwriting might display chaotic dynamics. Over recent years there has been an increasing interest in the application of dynamical analysis techniques to physiology and neurology, with mixed results. Grassberger and Procaccia (1983) developed a simple algorithm for the calculation of the number of dimensions needed to explain the behavior of a system, based on earlier work by Packard, Crutchfield, Farmer and Shaw (1980) and Takens (1981). This D_2 algorithm has been used extensively, for example in the study of EEG activity during sleep and while awake (Babloyantz, 1985) as well as during epileptic seizures (Babloyantz & Destexhe, 1986). In these and later studies of EEG recordings it was concluded that the dynamics underlying certain instances of brain activity may be generated by low dimensional deterministic chaotic attractors (Babloyantz, 1985, 1991; Babloyantz & Destexhe, 1986; Babloyantz & Lourenço, 1994). While some of the earlier findings have been criticized for their use of the Grassberger and Procaccia (1983) D_2 algorithm when their data was not adequately stationary (Mayer-Kress et al., 1988; Skinner, Molnar & Tomberg, 1994), they do provide some evidence that the neurological and physiological processes may involve chaotic dynamics.

Dimensionality analysis has also been applied to the study of motor skills. Kay (1988) for example, used dimensionality analysis in an attempt to measure the number of degrees of freedom produced during a simple rhythmic finger movement. Kay noted that while there are difficulties involved in the computation of dimensionality estimates, this type of analysis could produce useful information. It was concluded that the simple rhythmic finger movement was at least low dimensional (mean correlation dimension = 1.165, SD = 0.068).

The number of dimensions calculated for a movement gives an insight to the minimum number of dynamical degrees of freedom of the system. Kay (1988) writes that since we assume that our systems are nonconservative, the actual number of control parameters (degrees of freedom) will be the same or greater than that found in dimensionality analysis. What we measure is simply the end product of the process whereby the infinite possible degrees of freedom are constrained into a small number of functional synergies (Bernstein, 1967; Kay, 1988; Kugler, Kelso & Turvey, 1980; Kugler & Turvey, 1987; Mitra et al., 1998; Saltzman & Kelso, 1987).

A more recent study of simple rhythmic movements involved participants manipulating a pendulum of varying rotational inertia, rotating about the center of the wrist at a frequency they felt was natural (Mitra et al., 1997). Large and small virtual limbs were simulated using this apparatus. Mitra et al. note that simple rhythmic movements such as locomotion are usually modeled as limit cycle oscillators and that deviations from a single curve in the phase plane of position and velocity are assumed to be a result of stochastic physiological noise. They propose as an alternative to this that the movements are generated by higher dimensional nonlinear oscillators that are potentially chaotic.

Analysis of the rhythmic movements found that low dimensional attractors could explain movement variability, rather than the high dimensional attractors expected from stochastic noise. These attractors were found to display a positive largest Lyapunov exponent, a negative sum of Lyapunov exponents and low, fractional Lyapunov dimensions. These are necessary conditions for chaotic dynamics, as will be discussed below. Mitra et al. (1997) conclude that within the limits of current methods, the results provide strong evidence that these simple rhythmic movements are produced by a chaotic evolution on a strange attractor. As an extension of this work, Mitra et al. (1998) discuss intermediate motor learning as decreasing (dynamical) degrees of freedom. The utility of chaos analysis in movement research was once again demonstrated with the finding that as a skill is learnt, the dimensionality of the movements gradually reduce to a point where the participant becomes relatively proficient.

More specific to the study of graphic skills, the fractal nature of handwriting has been investigated by Dooijes and Struzik (1994). Fractional, or fractal, dimensionality is a well-known property of chaotic systems. The dimensionality of handwriting was calculated in its static form using the box counting method and in its dynamic form using the divider algorithm. It was tentatively concluded that the dimensionality of handwriting was fractional (between 1 and 2), despite there being intrinsic difficulties in calculating its *exact* value using these methods. It is important to note that it is the simple loop-like rhythmic movement patterns that are thought to be fractal, not the static letter pattern itself. This finding suggests that handwriting dynamics may be produced by a low, fractionally dimensional system, but cautions the experimenter from using these methods for the calculation of dimensionality estimates.

Handwriting dynamics are in general thought to be produced by the coupling of velocity (or force) generating oscillators (Dooijes, 1983; Hollerbach, 1981; Maarse, van Galen & Thomassen, 1989; Plamondon & Clement, 1991; Wann & Nimmo-Smith, 1991). Hollerbach (1981), for example, has proposed that the mechanism that produces the oscillation patterns in handwriting is the spring muscle model, which is based on a simplification of the length–tension curves of muscles. This length–tension relationship is a classical and robust finding in the biomechanical literature.

Biomechanical research has found that generally a muscle can only achieve maximum tension if it is at its optimal length (Armstrong, Huxley & Julian, 1966; Luttgens & Wells, 1982). At lengths greater or less than this optimum length, the tension generated will be lower. As a muscle contracts from its greatest length, the possible tension increases until it reaches its maximum, then drops again until the muscle is fully contracted. This relationship, known as the length–tension curve, was discovered by measuring the variation in tension as the length of the muscle fibers changed. Muscle fibers (sarcomeres) were stimulated to develop tension under constant contraction while the muscle length was maintained (i.e. isometric contraction). These length–tension curves have been used to investigate tremor oscillations produced in muscles under constant contraction (Akamatsu, Hannaford & Stark, 1986).

Akamatsu et al. (1986) studied the stability properties of sustained muscle contraction at the level of individual sarcomeres. Two components are involved in the relationship between length and tension. The first is a passive elastic relation that acts instantaneously, while the second is an active relation that occurs after a short delay. This results in a pause between the time at which a muscle changes its length and when it reaches its maximum tension at that length. A simulation model of this process leads to an inherent tremor mechanism as follows. The simulation begins at a point on the passive length–tension curve. When a constant contraction of the muscle occurs, the active component of the length–tension curve dominates the process. This leads to a new level of tension that must be compensated by a change in the length on the passive part of the curve. A change in the length of the muscle leads to a reactivation of the active part of the curve and then the process repeats.

This relationship leads to an iterative process that can be mathematically evaluated. Akamatsu et al. (1986) found that by increasing the value of the model's parameter related to activation (i.e. contraction), the output generated changes from stable convergent behavior to limit cycle oscillation and finally to chaos. These qualitative states were related to various types of muscle tremor found in the literature, particularly in isometric mammalian types of tremor and in some types of tremor corresponding to the open stretch reflex loop. It is worth noting that Akamatsu et al. merely demonstrated chaotic dynamics in a *model* of muscle tremor, not the muscle itself.

There is evidence that muscle tremor is an inherent mechanism which is filtered by mechanical properties of the limb under inertial loads as well as increasing joint stiffness due to muscle co-contraction (Joyce & Rack, 1974; Fox & Randell, 1970). Akamatsu et al. (1986) speculated that chaos serves possibly as the noise source that is filtered by the limb during movement.

Support for the hypothesis that tremor is an inherent noise source within the muscles has recently come from research into complex skills such as handwriting. Van Galen, van Doorn and Schomaker (1990) compared pen movements in line drawing and found that execution of more complex tasks disinhibited physiological tremor. This tremor was regarded as a potential measure of what they term neuromotor noise.

Van Gemmert and van Galen (1997) extended these findings and theorized that there is an intrinsic degree of random low level activation in the cognitive/biomechanical system, likening it to the physical equivalent of waste thermal warmth in a power plant. Cognitive noise can be raised, for example by increasing mental load through a secondary distracter task or by elevating processing demands by using more complex or difficult tasks. Within the biomechanical system this neuromotor noise may originate, at least in part, from muscle tremor. During the performance of skilled movements the accuracy of the output is affected by the signal level (the desired movement) relative to the level of background noise (neuromotor noise). In other words, performance accuracy is a function of the signal to noise ratio.

Van Galen, Portier, Smits-Englesman and Schomaker (1993) proposed that the motor system filters out neuromotor noise to produce movements that are kinetically optimal and spatially predictable. The more efficient the motor system, the more the noise is filtered and the less variable is the output. This filtering process is thought to be partly regulated by stiffening of the muscles and/or by an increase in friction, for example between the pen tip and the writing surface. As the computational demand of the motor task increases there is a decrease in the motor system's ability to filter out muscle tremor. These researchers concluded that an ability to minimize this inherent noisy variability is paramount for producing accurate movements.

Longstaff and Heath (1997) found evidence supporting the important role played by noise in motor performance by comparing the handwriting dynamics of people who were judged to be more proficient at the skill with those who were less proficient. We found that those participants who displayed less inter-trial variability produced the more legible output. We concluded that spatial variability in a stable physical environment is partly due to dynamic variability, spatial accuracy depending on a minimization of unwanted dynamic noise.

This evidence from the handwriting literature, when seen in the light of findings from biomechanics, suggests that the psychomotor system functions by minimizing the impact of muscular noise on movement outcomes. The primary source of this noise is an innate muscular tremor that is filtered via the limb mechanics during movement. The noise level can be modulated using strategies such as increasing limb stiffness or increasing friction. A failure to effectively limit the amount of noise leads to a loss of performance accuracy.

The question to be answered then is whether this noise is actually random variation or results from deterministic chaos inherent in the system's dynamics, as proposed by Akamatsu et al. (1986) and Mitra et al. (1997, 1998). This paper gathers further evidence supporting the proposal that what appears to be random variation within the motor output is in fact chaotic variability. The rest of this paper is organized as follows. A brief outline of chaos theory will be introduced along with a general description of some nonlinear data analysis techniques. A detailed explanation and justification of the methodology will then be presented, taking into account the problems and assumptions associated with these techniques. After outlining the experimental methods used to collect handwriting data, we employ a novel nonlinear procedure to analyze the data. In the discussion we outline the significance of this analysis for both model building and the individual assessment of psychomotor skill.

3. Nonlinear dynamic systems theory

Physical processes such as handwriting which are commonly dynamic, or time dependent, are often modeled by systems of equations. In experimental or real world situations it is almost always the case that we do not have access to the actual equations governing a system. Instead we have a set of observations of the system as it changes over time. This set of observations is called a time series. The goal of the researcher is to discover the set of equations that completely describes the process generating the time series in order to control the process and possibly predict future values of the system. Various time series analysis techniques have been developed to achieve this goal in the behavioral sciences (Gottman, 1981). However, there are some systems that are so complex that all that can be achieved is a more general description of some of the properties of the system under study.

The study of equations from known mathematical systems has shown that when the parameters of the equations governing a system vary due to situational or environmental circumstances, changes in the qualitative structure of the model's solutions may occur. Changes over time in the state of the system, represented by these solutions, can be represented graphically as a plot of key variables such as velocity and position in multidimensional space. The region of this space covered by all possible solutions is called the state, or phase, space. Over time the solution of a dynamic process moves through this state space along one of its trajectories.

An examination of the state space can inform us about the overall behavior of the system. For example the output of a process often contains equilibrium regions to which solutions of the system converge or are attracted. In these regions, no matter what the initial conditions are, the process will return to the same set of values.

There are four basic attractors (Poincaré, 1892, as cited in Shuster, 1988; Tsonis, 1992). The first is the fixed-point attractor in which case the dynamic process reaches a fixed point in the phase space and stays there. Since the state of the system at any time can be fully described by a single point, its dimensionality equals zero. The second attractor is the limit cycle. In this case, the solutions continually oscillate around the same set of values within the phase space. As a result of this the dimensionality of the limit cycle equals one.

The third attractor is produced when two or more oscillators are coupled together. Diagrammatically, the trajectories generated by this system resemble a two (or greater) dimensional torus, its shape looking like a doughnut. The solution of the system will completely fill the torus if the ratio between their frequencies is irrational (called quasi-periodic) and will merely travel around the edges if it is rational (periodic). When the coupling is weak, quasi-periodic solutions dominate, whereas when the coupling is strong, periodic solutions occur which indicate phase locking.

These three attractors all occur in stable state systems. The fourth type of attractor, called a chaotic or strange attractor, occurs in quasi-stable systems. In this case, the phase trajectories are confined to a finite space, but local points diverge. This is described as stretching and folding of trajectories. Chaotic systems are characterized by complex solutions and dynamics but are the result of simple deterministic equations. The output of a chaotic system is so complex that it appears random but has some short term predictability (Ott, 1993).

There are several important techniques for determining which of these attractors are characteristic of experimental time series. The techniques used in this paper, such as the study of delay portraits, calculation of correlation dimensions, calculation of Lyapunov exponents and examination of surrogate data will now be discussed.

3.1. State space diagrams and delay portraits

A plot of important system variables can tell us a lot about that system's dynamics. However, in experimental time series we do not have access to the various variables we may wish to plot. What is needed is a way of reconstructing the dynamics of the system from observables. One solution to this dilemma, developed by Packard et al. (1980), Ruelle (1981) and Takens (1981), is to generate a topological equivalent to the state vector $X(t)$ by taking the observable $x(t)$ as the first coordinate, $x(t+\tau)$ as the second and $x(t+(n-1)\tau)$ as the last. τ is the delay parameter and n is the embedding dimension. The lag is often chosen using known properties of the system, or to maximize the structure in the resulting plot. The embedding lag should be chosen so that each successive point is independent from previous points, for example by making sure they are decorrelated (Takens, 1981; Tsonis, 1992).

The effect of this embedding technique is to transform the unidimensional time series into a sequence of vectors in an n-dimensional space. To achieve accurate results, the embedding dimension, n, needs to be chosen so that $n \geqslant 2m+1$, where m is the dimension of the underlying attractor. The result of this method is known as a delay plot. It has been demonstrated that if suitable embedding dimensions and lags are used, this technique is mathematically equivalent to what you would achieve if you had plotted the known system variables (Takens, 1981; Tsonis, 1992).

An examination of the delay plot helps discriminate between the four bounded attractors, as well as random variation. Given error free data, after the solutions of the system stabilize, a point attractor will simply look like a point, a limit cycle attractor will look like an ellipse in two dimensions and coupled oscillators will look like a doughnut in three dimensions. A random time series will not display any complex structure. Finally, a strange attractor will produce a complex noncontinuous pattern, which we term fractal. In experimental time series, which are undoubtedly contaminated with noise, these plots will contain random variation that blurs the structure. However, if the signal to noise ratio is large enough, these plots provide a very useful qualitative method for discriminating between the various possible attractors.

3.2. Correlation dimension

A quantitative method for determining which attractor might be characteristic of a given real world process involves computing the system's dimension. The dimension of a system refers to the minimum number of scalar variables needed to model the dynamic process, or contain the attractor and hence provides a measure of the system's complexity. A commonly employed measure is the correlation dimension, D_2, which is based on geometric properties of the attractor in phase space.

D_2 is defined by Eq. (1) (Grassberger & Procaccia, 1983).

$$D_2 = \lim_{\varepsilon \to 0} \frac{\log C(\varepsilon)}{\log(\varepsilon)}, \tag{1}$$

where the correlation integral, $C(\varepsilon)$, is defined by

$$C(\varepsilon) = \lim_{N \to \infty} \frac{1}{N^2} \sum_{i,j=1}^{N} H\left(\varepsilon - \left|\boldsymbol{x}_i - \boldsymbol{x}_j\right|\right). \tag{2}$$

In Eq. (2) N is the total number of points, the ith point being represented by the m-dimensional vector, $\boldsymbol{x}_i$. $H(\cdot)$ is the Heaviside function which equals 1 when its argument is greater or equal to 0, and is equal to 0 otherwise. Eq. (2) counts the number of pairs of points that are no greater than ε apart as a proportion of the total number of pairs of points in the data set. In practice, for any fixed value of embedding dimension, D_2 is estimated as the average slope in a plot of $\log C(\varepsilon)$ against $\log(\varepsilon)$ within a central almost linear scaling region.

In stable systems, estimates of D_2 will level off at a particular value as the embedding dimension increases. The correlation dimension for stable periodic system will be a small integer, while the value for a chaotic process is a small nonintegral fractional value, i.e. it quantifies a fractal process. A completely random process is infinitely dimensional, the calculated dimension increasing as the attractor is embedded in increasingly larger dimensions. In practice, since we are dealing with finite time series at low embedding dimensions the asymptote value of D_2 for a random time series will be some finite number not much larger than 6 or 7.

3.3. Largest Lyapunov exponent

The second quantitative measure to be examined is the Lyapunov exponent spectrum. In a given embedding dimension, the Lyapunov exponent is a measure of the speeds at which initially nearby trajectories of the system diverge. There is a Lyapunov exponent for each dimension of the process, which together constitute the Lyapunov spectrum for the dynamical system. The Lyapunov exponent is related to the predictability of the system. The largest Lyapunov exponent of a stable system does not exceed zero, while a chaotic system has at least one positive Lyapunov exponent. A random system is completely unpredictable and therefore the largest Lyapunov exponent theoretically should be infinitely positive. In practice the calculated largest Lyapunov will be a large positive number for noisy processes.

A positive Lyapunov exponent implies that a chaotic process displays long term unpredictability, with the output being sensitively dependent on the initial conditions. Even slightly different initial values can lead to vastly different system outputs. The sum of all Lyapunov exponents of a chaotic system will be negative, consistent with the idea that the chaotic attractor is globally stable. The more positive the largest Lyapunov exponent the more unpredictable the system. When the equations governing a system are known, the definitive test for chaos is one positive Lyapunov exponent with a negative sum of Lyapunov exponents. However due to the impact of noise within experimental systems, this is not a reliable test for chaos unless some attempt is made to minimise measurement noise. While a failure to satisfy this criterion in an experimental time series is a good argument for it not being chaotic, merely satisfying this criterion is not sufficient to determine that it is chaotic. A detailed description of the calculation of the Lyapunov spectrum is beyond the scope of this paper and the reader should refer to Tsonis (1992) or Wolf, Swift, Swinney and Vastano (1985), for example. It is worth noting that Akamatsu et al. (1986) found a maximum Lyapunov exponent of 0.22719 bits per iteration (i.e. log base 2) for their muscle model while it was within its chaotic state.

4. Description and justification of methodology

4.1. Concatenation of data sets from individual trials

There are several problems associated with nonlinear dynamic analysis. The accuracy of the results of these calculations usually depends on the availability of large data sets. There is debate as to the minimum number of data points necessary with some recommendations exceeding 10 000. Recent research has found that accurate results can be achieved with as few as 1584 data points [2] when the dimension of the system (d) is 3. Even this value is thought to be an overestimate for some systems and in practice smaller sized data sets might suffice, particularly when rhythmic behavior is being studied (Kay, 1988).

Unfortunately there is no standard method for determining the errors of estimates of quantities such as D_2 and the maximum Lyapunov exponent for all types of systems. Although less than 1500 data points may be needed in the present study, a typical handwriting trace sampled at a frequency of 100 Hz will only result in a time series containing between 300 and 500 data points. Because of this concern with an adequate data sample for nonlinear analysis, we examined the possibility of concatenating data obtained from five successive handwriting trials. Typically this new time series will contain more than 1500 data points. Our first step is to determine whether this technique will produce theoretically and mathematically valid results.

The calculation of Lyapunov exponents and D_2 requires assumptions that the data points comprising the time series are temporally related and sampled at equal time intervals. The question we answer in this section is whether we obtain equivalent results by joining velocity profiles to that

[2] A recent estimate of the number of data points needed is $N = 10^{2+0.4d}$ i.e. 1584 for $d = 3$ and 3981 for $d = 4$ (Nerenberg & Essex, 1990; Tsonis, 1992).

which we would achieve by simply using each individual time series and computing averages of D_2 and the Lyapunov exponents. From an experimental point of view, it would be difficult at best for a participant to write the same word five times continuously and without pauses due to gross movements of the arm and limitations on the size of the writing tablet. Furthermore, this strategy would increase the likelihood of introducing nonstationarity into the data due to some combination of fatigue and pausing during a long arm motion. Nonstationarity in the time series would further complicate the data analysis.

Combining the data from consecutive trials seems reasonable from a theoretical point of view. We propose that writing the same word several times can be likened to taking several samples from the same stable attractor. Since the analysis techniques are designed to find the properties of this attractor, each of the time series should produce similar results. When combined they should produce a more accurate portrait of this attractor. If the samples are from a point attractor, or simple limit cycle periodic system, the combined data will produce similar results to the original. If the samples are chaotic, each trial is merely a sample from the same strange attractor. If the original time series is 'noise', then the joined series will also be noise. If the time series for each individual 'chunk' is not a sample of the same stable attractor then the results for each chunk will be substantially different from each other and presumably the concatenated time series will be quite disordered. The consequence of having the 'chunks' of data temporally disjointed in any of these cases will be merely to introduce a trivial number of short 'noisy' sections where the samples are joined. The concatenation of short sequences of data before data analysis is a commonly used technique in EEG research (Barlow, Creutzfeldt, Michael, Houchin & Epelbaum, 1981) as well as in the spectral analysis of handwriting dynamics.

One way to test the validity of this methodology experimentally is to compare the dynamic characteristics of the five original time series against those of the five concatenated time series. If there is no difference between them in estimates of D_2 and the maximum Lyapunov exponent, it is reasonable to conclude that the methodology is valid.

4.2. Noise reduction using singular value decomposition

One overriding issue of concern in nonlinear dynamic analysis is the effect of 'noise' on the accuracy of the results. Chaos is related to noise in the sense that to a large extent chaotic data display many of the characteristics of noise, such as a broad band power spectrum and a rapid decline in the prediction accuracy over time. Experimental time series are typically contaminated with noise. Since noise is infinite dimensional, if a time series is contaminated with even a small amount of noise, the data analysis can produce correlation dimensions larger than that of the true attractor (Tsonis, 1992). Noise will also tend to produce positive Lyapunov exponents even when such values do not exist in the deterministic component of the true series (Sprott & Rowlands, 1995). It is essential therefore to eliminate, or at least minimize, the amount of noise within the experimental data sets. A useful method of noise reduction is singular value decomposition (SVD), a technique recommended and commonly used in nonlinear data analysis (Aubry, Holmes & Lumley, 1988; Lumley, 1970; Rapp, 1994; Rowlands & Sprott, 1992; Sauer, 1992; Sprott & Rowlands, 1995).

SVD derives orthogonal eigenfunctions from the original time series, each of which corresponds to one of a sequence of decreasing eigenvalues. Each eigenvalue represents the relative contribution of the corresponding eigenfunction towards fitting the original time series. A new time series based on the first few eigenfunctions weighted by their respective eigenvalues is computed. Since the additive noise is distributed evenly across the eigenfunctions the SVD method serves to increase the signal–noise ratio without altering the basic properties of the attractor.

5. Using surrogate data to confirm that handwriting velocity profiles are different to noise

Since chaos has similar properties to noise we need to test that the time series is not simply a random process. We do this by firstly generating surrogate time series from the original data and then comparing the results of the chaos analysis of the surrogates with those obtained from the original time series analysis (Theiler, Eubank, Longtin, Galdrikian & Farmer, 1992). These surrogate time series contain the same numerical values as the original time series but they have been randomly mixed in one of several ways in order to remove their deterministic recursive structure. If the results are similar for the original and surrogate series, then the original time series is most likely noise, or heavily contaminated with noise. If the nonlinear dynamic analysis produces different results for the two types of time series then it is likely that the original time series was nonlinear and deterministic, i.e. chaotic or possibly periodic.

There are three main types of surrogate data sets (Rapp, 1994; Theiler et al., 1992). The first is simply a random shuffle of the data and tests the null hypothesis that the original time series is no different from uncorrelated noise. Applying a Fourier transform to the data, randomizing the phases and then applying the inverse Fourier transform produces the second surrogate. This process removes temporal dependencies in the data while maintaining its spectrum. This is a test of the null hypothesis that the original experimental time series is the same as linearly autocorrelated Gaussian noise. The third surrogate is a generalization of the second surrogate to non-Gaussian noise. The time-order of the series is shuffled while maintaining the linear correlations of the original time series. This procedure tests the hypothesis that the original time series is no different from a static nonlinear transform of linear Gaussian noise.

The aim of this study was to demonstrate the validity of the preprocessing techniques of data concatenation and noise reduction and then to gather evidence for the proposal that handwriting dynamics are chaotic. The present study initially compares the results of a nonlinear dynamic analysis of time series both in their original form and after they have been chunked to simulate five temporally disjointed samples of an attractor which have been concatenated. The time series used are two known chaotic series and one random series. It is hypothesized that the chunking and concatenation of the time series will not substantially alter the calculated dynamic parameters of these systems. For the known chaotic systems, this procedure will not destroy the structure of the attractors and for the random time series no structure will be introduced.

The results of the nonlinear dynamic analysis of velocity profiles for five handwriting samples will be compared with those for time series generated by the concatenation of these five samples for two participants. It is hypothesized that the attractors for each of the five trials of handwriting will be fundamentally similar to each other and to the five trials concatenated, both visually and in terms of the correlation dimension.

It is hypothesized that a chaos analysis of the concatenated velocity profiles will generate significantly different results for surrogate data sets based on these original time series. In particular, the phase portraits will show less structure and the correlation dimension estimates will be higher. Finally, it is hypothesized that the correlation dimensions will be small (i.e. between 0 and 5), that there will be at least one positive Lyapunov exponent and that the sum of Lyapunov exponents will be negative, providing evidence for the proposal that handwriting velocity profiles are chaotic.

6. Method

6.1. Participants

Eight participants aged between 19 and 41 voluntarily consented to participate in this study. All participants were recruited from the general university population. No special criteria were used to select participants, however all participants wrote naturally with their right hand and had normal or corrected-to-normal vision. This experiment was carried out according to the ethical guidelines laid down by the University of Newcastle's Human Research Ethics Committee.

6.2. Apparatus

The data were collected using a Wacom 1212-R graphics tablet connected to a Macintosh IIci personal computer. This setup recorded the X and Y coordinates of the tip of the stylus as it moved across the tablet surface. The coordinate data were sampled at a frequency of 100 Hz, and at a spatial resolution of 0.02 cm. The stylus used was similar in size and weight to a normal ballpoint pen. A piece of plain white paper was secured onto the face of the tablet to provide a more natural writing surface. A single horizontal line on the page was the only directional guide provided.

6.3. Procedure

The participants were seated at a desk of typical height (60 cm) which contained both the computer and the graphics tablet. The participants were asked to write the pseudo-word 'madronal' in their normal cursive script. This pronounceable pseudo-word created by the experimenter satisfied the criteria that the pen does not need to leave the page during writing, that successive letters were not too similar in form and that the resulting pen trace contained at least 300 samples.

The participants were allowed to practice the task until they felt comfortable with what was required. The participants were then asked to write the pseudo-word ten times in their normal cursive handwriting on the graphics tablet using the stylus. There was no immediate visual feedback to the participants of their pen trace.

7. Results

7.1. Preprocessing

Of the ten trials, the first trial was generally discarded as a practice trial and of the remaining nine trials only the best five were used for further analysis. The best trials were chosen on the basis of their being the first five trials not interrupted by mistakes or pauses uncharacteristic of the participant's general writing style. The data from the five handwriting trials were split into time series for the horizontal (X) and vertical (Y) directions. The basic velocity profiles were calculated by taking the first difference of the positional data.

For each participant the five velocity profiles were concatenated for both X and Y velocities. SVD was then applied to all velocity time series to reduce noise. Chaos data analyzer (CDA): The professional version (Sprott & Rowlands, 1995) was used for the calculation of the SVD, correlation dimensions and calculation of the largest Lyapunov exponent. Lyapunov spectra were calculated using the NETLE software (Gençay & Dechert, 1992; Kuan & Tung, 1995)

7.2. A comparison of the chaotic and random time series in their original and chunked forms

Whenever a new analysis technique is proposed, the usual procedure is to compare the results of the new analysis with those of previously used techniques. In the present case, two time series known to be chaotic and one representing noise were analyzed both in their original form and after they have been 'chunked' to simulate five temporally disjointed samples of the chaotic attractor. This procedure involved inputting the time series into a text editor and manually removing 4, 20–25 point sections quasi-randomly from approximately equally spaced intervals along the series. The two chaotic time series used are 2000 points sampled at time delays, $\Delta t = 0.05$, of the variable $X(t)$ of the Lorenz system Eqs. (3)–(5) and 2000 iterations of the variable X_n of the Hénon attractor (Eqs. (6) and (7)). The noise time series used was 2000 random data points with a Gaussian distribution which had a mean of zero and a standard deviation of one. These series were provided by the CDA computer program.

Lorenz: $a = 10$, $b = 28$, $c = 8/3$,

$$\frac{dX}{dt} = a(Y - X), \tag{3}$$

$$\frac{dY}{dt} = bX - Y - XZ, \tag{4}$$

$$\frac{dZ}{dt} = XY - cZ. \tag{5}$$

Hénon: $a = 1.4$, $b = 0.3$,

$$X_{n+1} = 1 - a\, X_n^2 + Y_n, \tag{6}$$

$$Y_{n+1} = b\, X_n. \tag{7}$$

A comparison of the properties of the original and chunked time series will tell us if this procedure alters the dynamics dramatically. Two methods will be used to assess the dynamics, the first being a qualitative examination of the delay plots, the second being a quantitative examination of the correlation dimensions. Figs. 1(a) and (b) show the delay plots for the original and chunked Lorenz series. A lag of 2 is used to best display the system dynamics. These plots show that, apart from a few 'noisy' points, the two delay portraits are identical. These noisy points arise from those few points in the time series that are temporally disjointed where the time series are concatenated.

Similarly, Figs. 2(a) and (b) show that for the Hénon time series, the only difference between the original and concatenated series is a few noisy points. A lag of 1 was chosen to display the dynamics in their traditional form. Finally, Figs. 3(a) and (b) show that after chunking the random (noise) time series remains completely unstructured. A lag of 1 was arbitrarily chosen for this plot. Since it is a random time series, the delay plot would produce a similar shape irrespective of the lag chosen.

Table 1 provides a comparison between the correlation dimension and largest Lyapunov exponent for the Lorenz, Hénon and random (noise) time series in both the original and chunked forms. The expected values of these parameters from the literature are also presented. As can be seen, there is little change in these important properties of the time series after chunking. In particular, correlation dimensions remain fractional and the largest Lyapunov exponent remains positive for the known chaotic time series. The

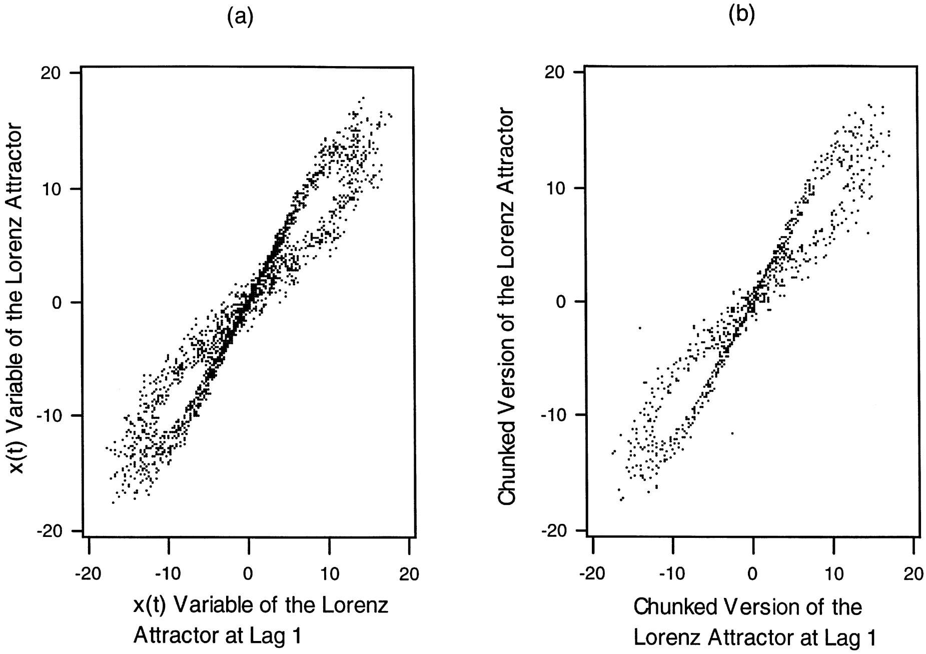

Fig. 1. (a) A delay plot of the variable $X(t)$ of the Lorenz attractor; (b) A delay plot of the chunked version of the variable $X(t)$ of the Lorenz attractor. The chunked time series was created by removing 4, 20–25 point sections quasi-randomly from approximately equally spaced intervals along the original time series. In both plots a lag of 2 provided the most structured attractor for the system.

correlation dimension remains substantially larger than 5 for the random (noise) time series. Note that there are too few data points to calculate accurate largest Lyapunov exponents for the Lorenz attractor.

7.3. Comparison of nonlinear dynamic analyses for five handwriting trials and for the five trials concatenated

The nonlinear dynamic characteristics of each individual handwriting trial were compared to the characteristics of the time series generated by concatenating five trials. The results for both the X and Y velocity profiles for two participants are presented below. No special criteria were used to select these participants for detailed study. As their results are typical of that found for all eight participants, it was not thought necessary to present the remaining six sets of results. Delay portraits are first examined to provide qualitative evidence, then the correlation dimensions are calculated to

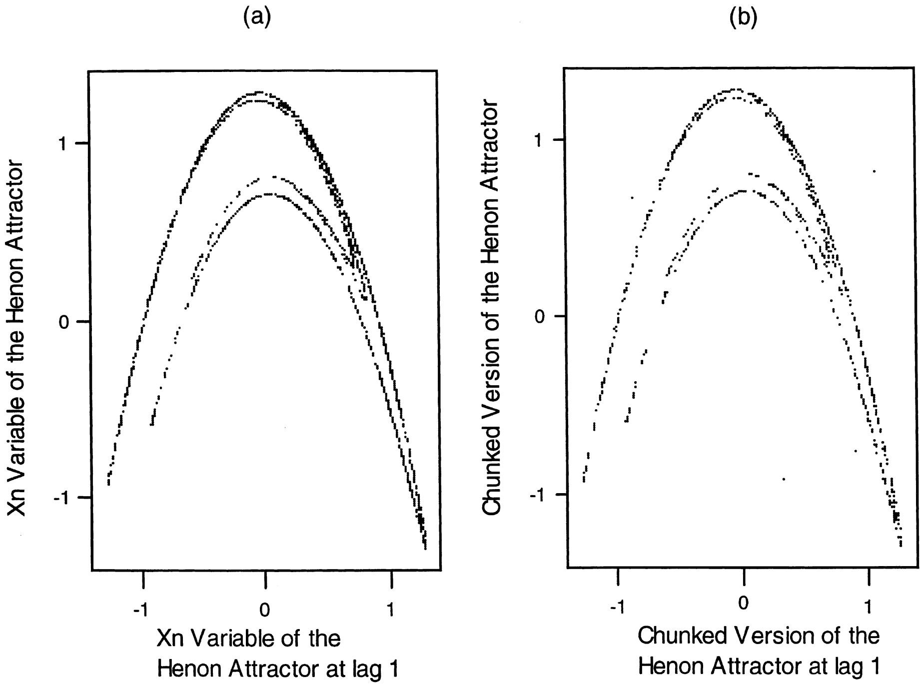

Fig. 2. (a) A delay plot of the Hénon chaotic time series; (b) A delay plot of the chunked version of the Hénon chaotic time series. The chunked time series was created by removing 4, 20–25 point sections quasi-randomly from approximately equally spaced intervals along the original time series. A lag of 1 was chosen for both plots to display the dynamics in their traditional form.

provide a quantitative comparison. There were too few data points in individual trials to calculate accurate Lyapunov exponents.

Figs. 4 and 5 display delay portraits of the X velocity for each handwriting trial as well as for the five trials concatenated for participants 1 and 8, respectively. Note that for both participants the individual attractors are similar for each trial as well as for the concatenated trials.

Table 2 contains the correlation dimension for the X velocity for each of five handwriting trials and for data obtained when observations from the five trials are concatenated for both participants 1 and 8. A two-tailed t-test was performed to confirm that the correlation dimension for the concatenated series was not significantly different from the correlation dimension of the constituent time series. The null hypothesis value for the population mean is assumed to be the D_2 estimate for the concatenated sample. A nonsignificant difference was verified for both participant 1 ($t(3) = -0.52$, $p = 0.63$), and

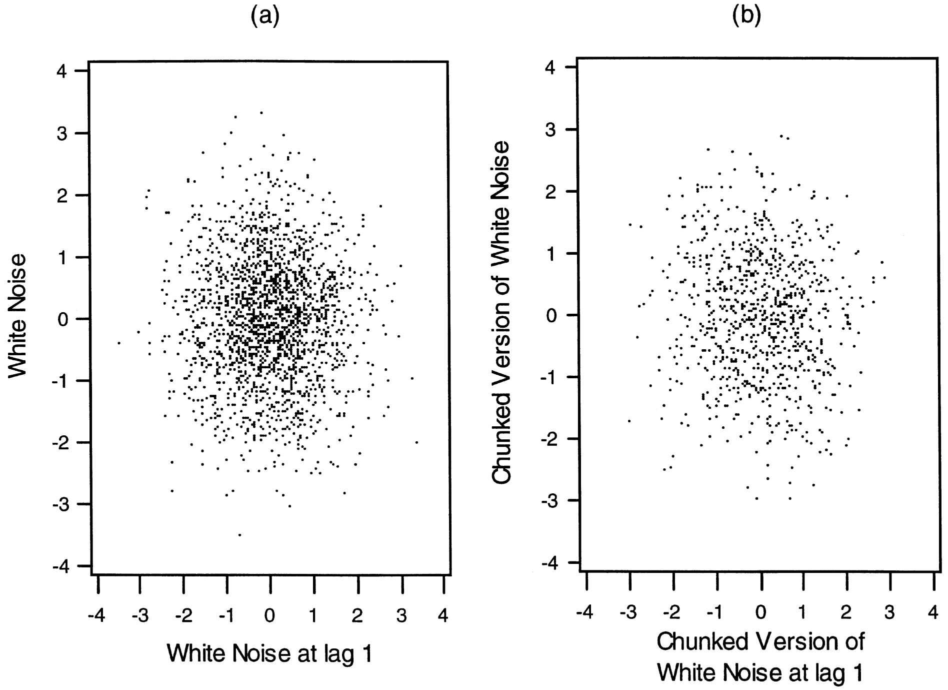

Fig. 3. (a) A delay plot of the random (noise) time series; (b) Delay plot of the chunked version of the random (noise) time series. The chunked time series was created by removing 4, 20–25 point sections quasi-randomly from approximately equally spaced intervals along the original time series. Since the time series is random, the delay plot would produce a similar shape irrespective of the lag chosen. For both plots a lag of 1 was arbitrarily chosen.

participant 8 ($t(3) = -0.68$, $p = 0.53$). Table 2 shows that both the individual time series and the concatenated time series display similar nonlinear dynamic characteristics.

Figs. 6 and 7 show delay portraits of the Y velocity for each handwriting trial as well as for the five trials concatenated for participants 1 and 8, respectively. Note that each individual attractor is similar for each trial as well as when the trials have been concatenated.

Table 3 contains the correlation dimension of the Y velocity for each of five handwriting trials as well as for the five trials after they have been concatenated. A two-tailed t-test was performed to check that the correlation dimension for the concatenated series was not significantly different from the correlation dimension of the constituent time series. This result was verified for both participant 1 ($t(3) = 0.20$, $p = 0.85$), and participant 8 ($t(3) = -0.97$,

Table 1
The correlation dimension and largest Lyapunov exponent of the Lorenz, Hénon and Noise data in their original and chunked forms

	n	Correlation dimension	Largest Lyapunov exponent [a]
Lorenz (expected)		Approx. 2.05 [b]	1.31 [c]
Lorenz (calculated)	2000	2.050 ± 0.112	N/A [d]
Chunked [e] Lorenz	1898	2.043 ± 0.117	N/A [d]
Hénon (expected)		1.21 [f]	0.602 [g]
Hénon (calculated)	2000	1.222 ± 0.056	0.597 ± 0.033
Chunked [e] Henon	1898	1.219 ± 0.069	0.613 ± 0.037
Noise (expected)		Greater than 5 [h]	$+\infty$ [i]
Noise (calculated)	2000	6.642 ± 6.642 [j]	0.780 ± 0.034
Chunked [e] Noise	1898	6.604 ± 6.604 [j]	0.787 ± 0.034

[a] Bits (factors of 2) per data sample (unit time) i.e. log base 2.
[b] Sprott and Rowlands (1995).
[c] Sprott.physics.wisc.edu/chaos/lorenzle.htm. The natural log for the Lyapunov exponent is 0.906.
[d] Too few data points were used to calculate accurate largest Lyapunov exponents for the Lorenz attractor (Sprott and Rowlands, 1995).
[e] The chunked data was produced by removing small sections from the original data to split it into temporally disjointed segments. These were then concatenated into one time series. This simulates five temporally disjointed samples of an attractor.
[f] Grassberger and Proccaccia (1983).
[g] Wolf et al. (1985).
[h] Sprott and Rowlands (1995).
[i] Sprott and Rowlands (1995).
[j] This error term means that D_2 is essentially unbounded.

$p = 0.39$). This result shows that both individual trials and the five trials concatenated display similar nonlinear dynamic characteristics, as was the case for the X velocity time series.

One crucial step in the calculation of nonlinear properties of a time series is the selection of the time delay. Before a result can be accepted, the analysis should be performed on the time series using a number of different delays in order to determine the sensitivity of the original results to this parameter. To investigate the validity of the results for the concatenated time series, the correlation dimensions were recalculated using delays of 2, 4, 8 and 16. The correlation dimensions calculated using these delays were comparable to the original results both for participant 1 (X velocity: mean $= 3.843$, SD $= 0.060$; Y velocity: mean $= 3.340$, SD $= 0.104$) and participant 8 (X velocity: mean $= 3.340$, SD $= 0.158$; Y velocity: mean $= 3.822$, SD $= 0.395$). Though these results were slightly higher than the original values, they confirm that the time series are low dimensional.

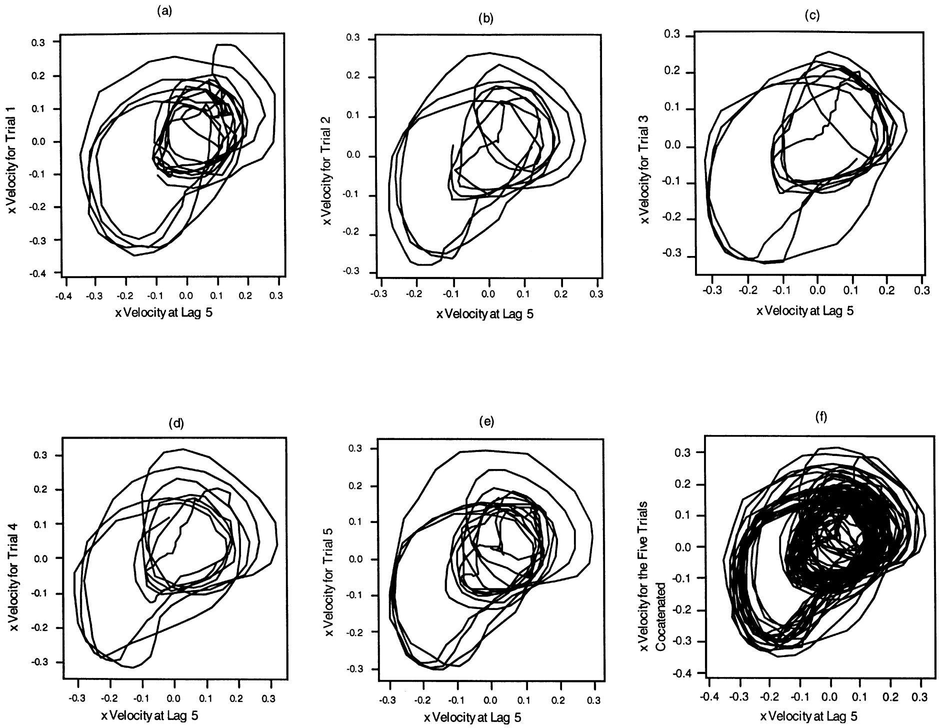

Fig. 4. A delay plot of *X* velocity for each handwriting trial (a–e) and the five trials concatenated (f) for participant 1. A lag of 5 was chosen to best display the dynamics of the system.

One final concern with using D_2 was that potential nonstationary sections in the data may have led to erroneous results. Nonstationarity is a well-known problem with physiological data (Mayer-Kress et al., 1988; Skinner et al., 1994). An alternative method of calculating dimensionality, called $PD2_i$ [3], was developed by Skinner et al. (Skinner et al., 1994; Skinner, Carpeggiani, Landisman & Fulton, 1991; Skinner, Goldberger, Mayer-Kress & Ideker, 1990; Skinner et al., 1990). This method is less sensitive to nonstationary sections within a time series. In order to ensure that this alternative method did not result in vastly different dimension estimates, it was applied to the concatenated time series for all eight participants. Once again the results were comparable for both *X* velocity (mean = 4.004, SD = 0.274)

[3] The $PD2_i$ software was used under license from Totts Gap Software, 1430 Totts Gap Rd Bangor, PA 18013, USA.

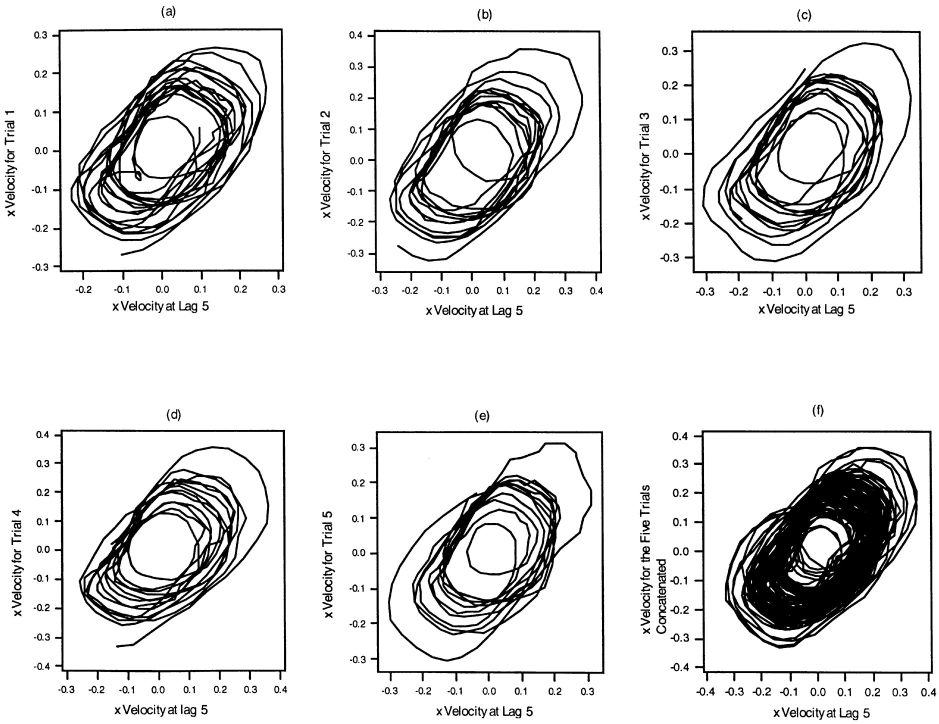

Fig. 5. A delay plot of X velocity for each handwriting trial (a–e) and the five trials concatenated (f) for participant 8. A lag of 5 was chosen to best display the dynamics of the system.

and Y velocity (mean = 3.961, SD = 0.255) time series. These values are also slightly higher than the original values, but further confirm that the handwriting velocity time series are low dimensional. Fluctuations in dimensional estimates within each time series were relatively small. There were no trends

Table 2
The correlation dimension of the X velocity for five handwriting trials and the five trials concatenated for participants 1 and 8

X velocity	Participant 1		Participant 8	
	n	Correlation dimension	n	Correlation dimension
Trial 1	360	3.533 ± 0.614	465	3.339 ± 0.519
Trial 2	231	3.499 ± 0.273	339	2.660 ± 0.390
Trial 3	234	3.479 ± 0.691	339	2.621 ± 0.236
Trial 4	237	2.654 ± 0.054	360	2.352 ± 0.671
Trial 5	360	3.257 ± 0.579	339	2.727 ± 0.352
Mean		3.284		2.940
5 Trials concatenated	1711	3.370 ± 0.506	2364	2.851 ± 0.374

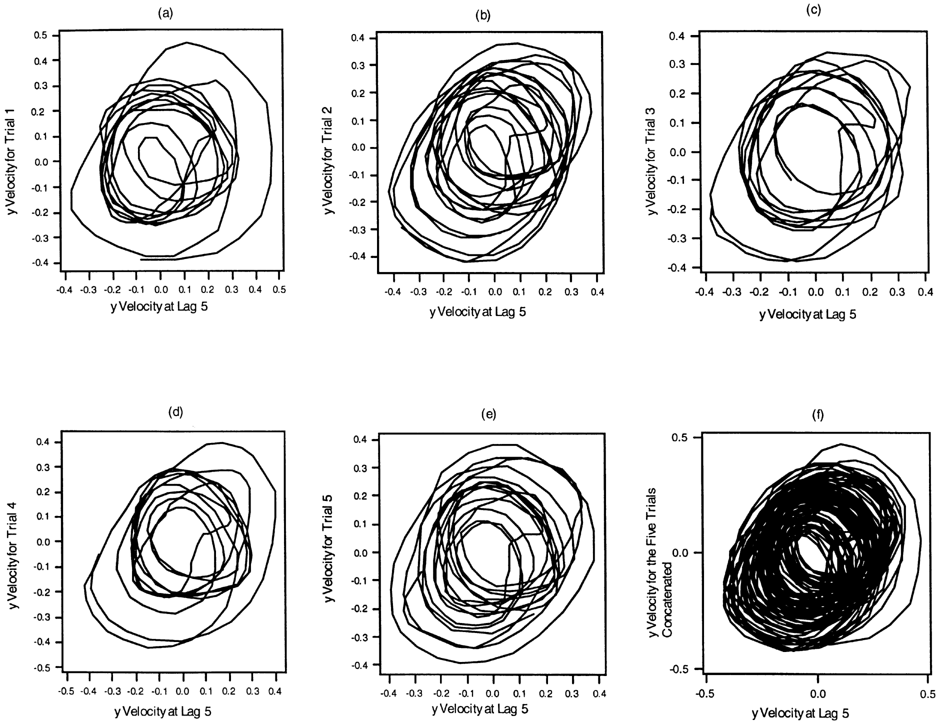

Fig. 6. A delay plot of the *Y* velocity for each handwriting trial (a–e) and the five trials concatenated (f) for participant 1. A lag of 5 was chosen to best display the dynamics of the system.

except for some local increases in the estimated dimension due to noise transients at the concatenation points. Since the results calculated for the dimensional estimates are similar when using different time delays as well as using different techniques, it was decided to remain with the original method of analysis.

7.4. Comparison of correlation dimension and largest Lyapunov exponent of X and Y handwriting velocity from five concatenated trials and their respective surrogates

Three surrogate data sets were generated from the concatenated handwriting velocity time series using the computer program MTRChaos (Rosenstein, Collins & DeLuca, 1993, 1994). These surrogate series are sequence randomized, phase randomized and Gaussian scaled versions of the

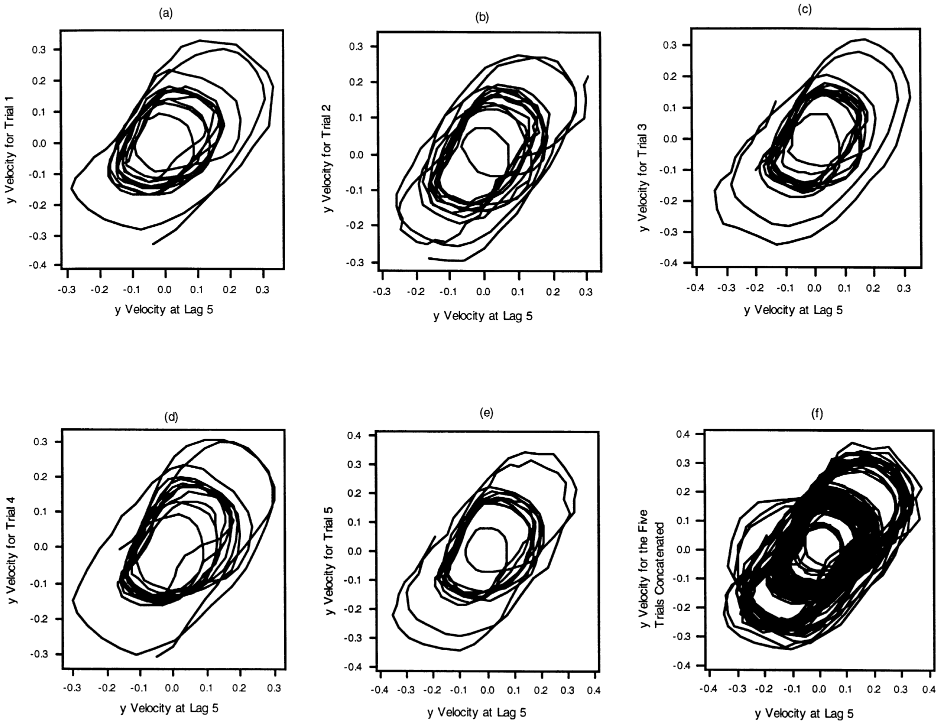

Fig. 7. A delay plot of Y velocity for each handwriting trial (a–e) and the five trials concatenated (f) for participant 8. A lag of 5 was chosen to best display the dynamics of the system.

original data. This comparative data analysis was performed to test whether the time series are actually uncorrelated noise, linearly autocorrelated Gaussian noise or a static linear transform of linear Gaussian noise. The

Table 3
The correlation dimension of the Y velocity for five handwriting trials and the five trials concatenated for participants 1 and 8

Y velocity	Participant 1		Participant 8	
	n	Correlation dimension	n	Correlation dimension
Trial 1	234	3.369 ± 0.315	342	2.692 ± 0.518
Trial 2	360	3.609 ± 0.659	387	2.537 ± 0.255
Trial 3	234	2.664 ± 0.083	366	2.820 ± 0.503
Trial 4	240	2.409 ± 0.247	360	2.611 ± 0.160
Trial 5	336	2.618 ± 0.108	339	2.578 ± 0.392
Mean		2.933		2.648
5 Trials concatenated	1714	2.886 ± 0.268	2308	2.696 ± 0.385

delay portraits and correlation dimensions are expected to be substantially different for the surrogates compared to the original.

Figs. 8 and 9 are delay portraits of the X and Y velocities and their respective surrogate time series for participant 1. Figs. 10 and 11 are delay portraits of the X and Y velocities and their respective surrogate time series for participant 8. In all cases it is apparent that the original time series' structure has been substantially altered when surrogate series are computed.

Figs. 8(b), 9(b), 10(b) and 11(b) show that the first surrogate, a sequence randomized version of the original time series, appears quite random (compare with Fig. 3). This indicates that the original time series was not merely an uncorrelated random time series. The phase portrait of the second surrogate can be seen in Figs. 8(c), 9(c), 10(c) and 11(c). This surrogate, which

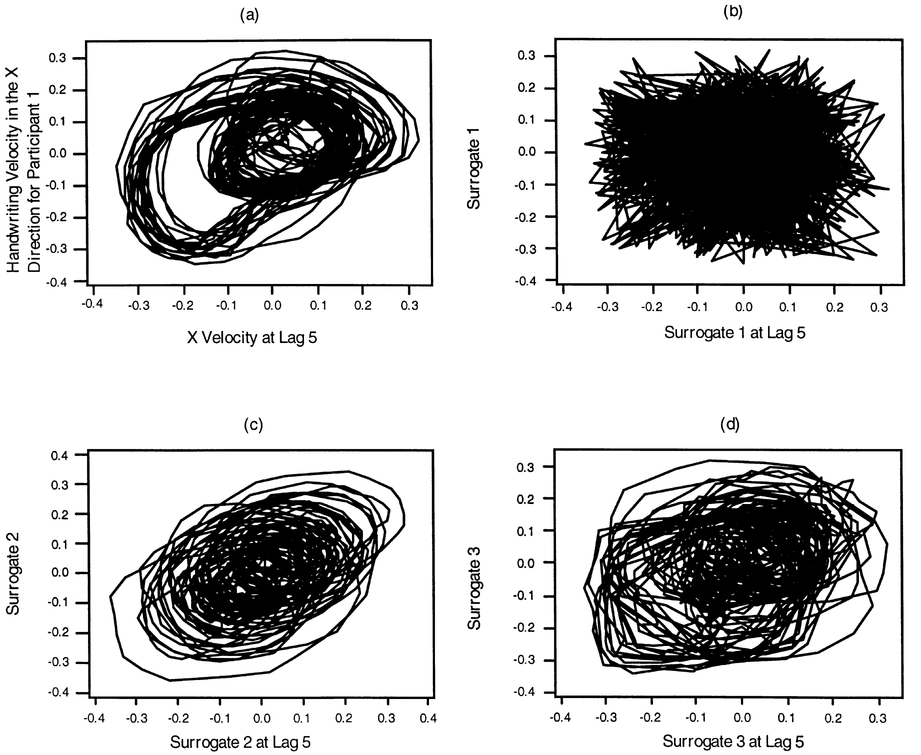

Fig. 8. Delay portraits of the X handwriting velocity (a) and its surrogate time series for participant 1. Surrogates (b), (c) and (d) are sequence-randomised, phase-randomised and Gaussian scaled versions, respectively. A lag of 5 was chosen to best display the dynamics of the system.

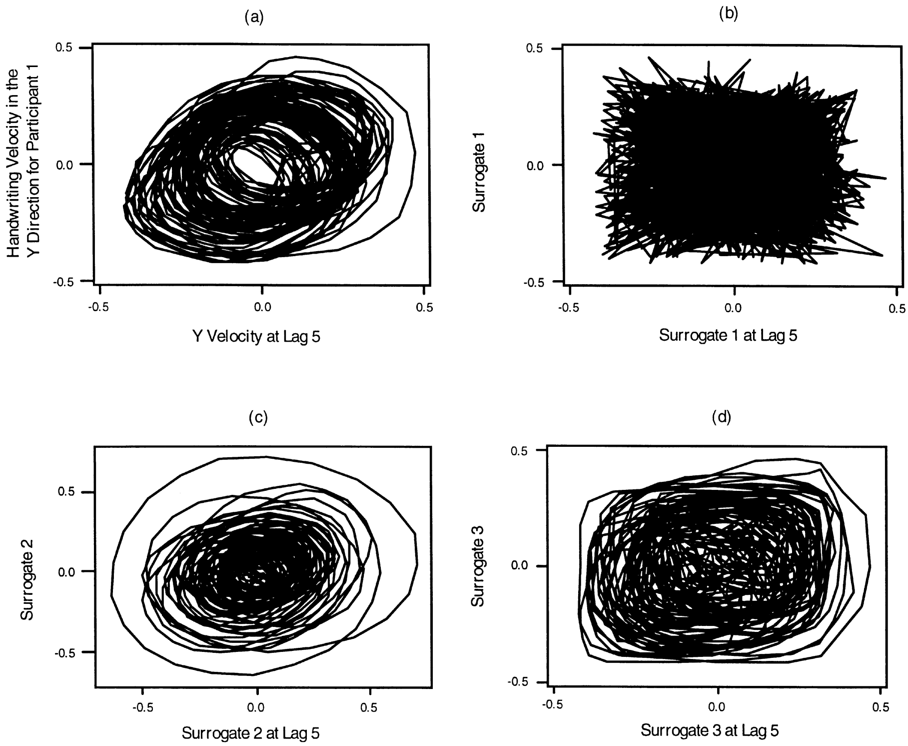

Fig. 9. Delay portraits of the *Y* handwriting velocity (a) and its surrogate time series for participant 1. Surrogates (b), (c) and (d) are sequence-randomised, phase-randomised and Gaussian scaled versions, respectively. A lag of 5 was chosen to best display the dynamics of the system.

retains the same frequency spectrum as the original time series but has had its phase randomized, demonstrates the persistence of the dominant frequency with the characteristic pattern being substantially altered.

The third surrogate, which can be seen in Figs. 8(d), 9(d), 10(d) and 11(d) is a generalization of the second surrogate to non-Gaussian noise and produces phase portraits with structure that is relatively dissimilar to that contained in the original series. When examining Figs. 8–11 it is apparent that each surrogate is more similar to other surrogates of the same type than to its base time series.

Table 4 shows that all surrogates display distinctly different properties to the original time series. The larger dimension values displayed by the surrogate time series indicate that they are not the same as the original. In the estimation of the correlation dimension, the surrogates did not appear to

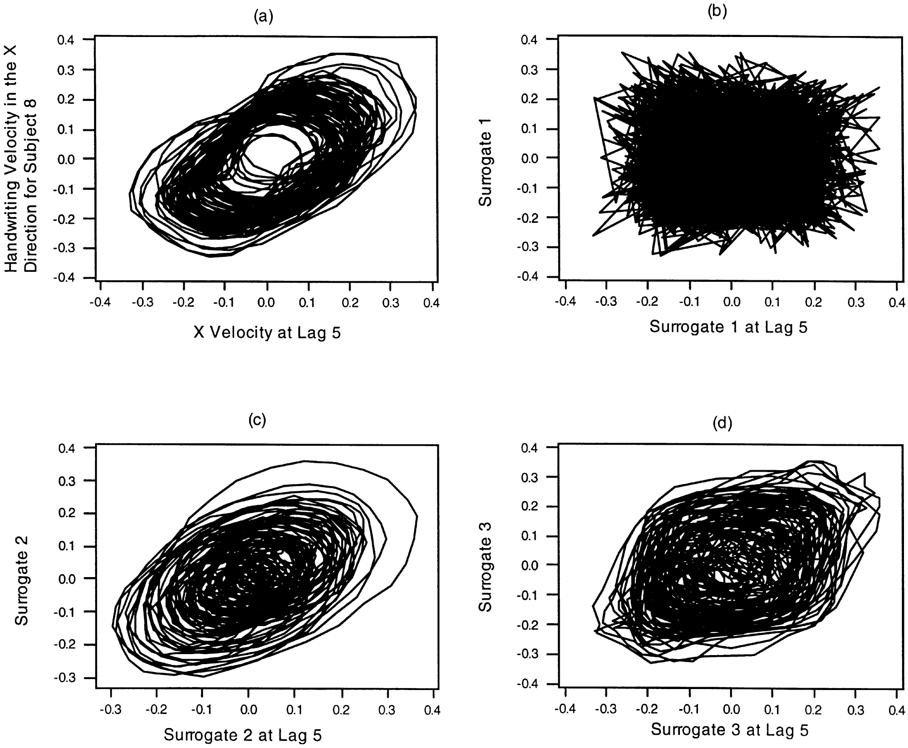

Fig. 10. Delay portraits of the X handwriting velocity (a) and its surrogate time series for participant 8. Surrogates (b), (c) and (d) are sequence-randomised, phase-randomised and Gaussian scaled versions, respectively. A lag of 5 was chosen to best display the dynamics of the system.

stabilize in less than ten embedding dimensions in the way that the original time series did. This further indicates a difference between the original time series and its surrogates.

The error term in Table 4 is a rough guide based on the way the correlation dimension is estimated (Sprott & Rowlands, 1995). Since the error term in the calculation for the X velocity for participant 1 is mildly large, there was some doubt that there was a significant difference between the value for this series and its surrogates. A more accurate method of testing if the original and surrogate time series are significantly different at an alpha level of 0.05 is to compute 19 surrogates and then show that the original time series value lies outside the confidence interval. For the X velocity of participant 1, a one-tailed t-test was performed comparing the correlation dimension for 19 versions of each surrogate with the correlation dimension of the original. In

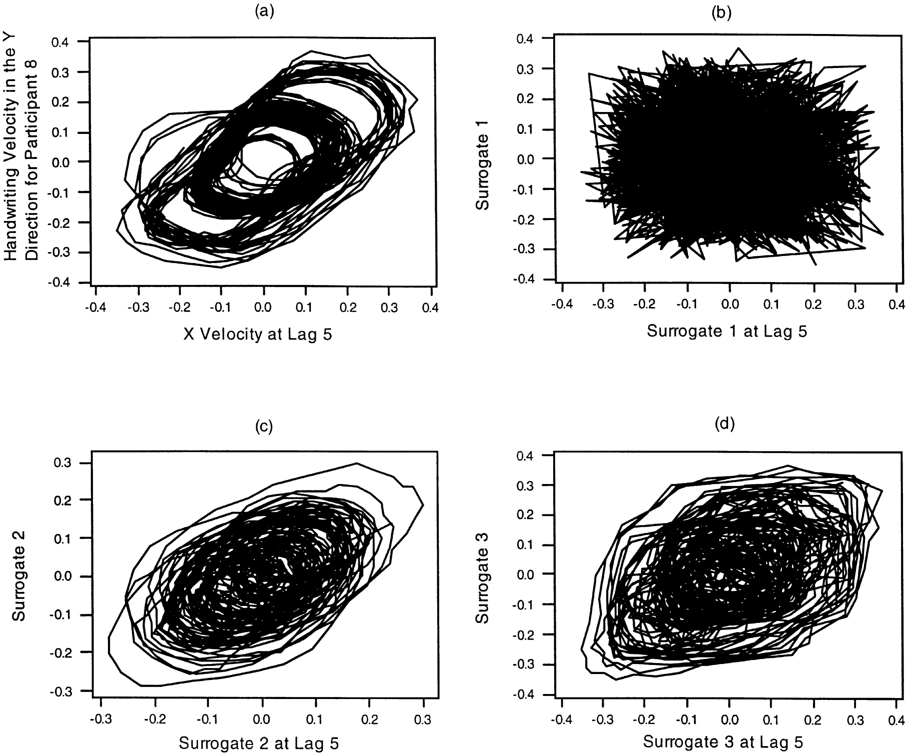

Fig. 11. Delay portraits of the *Y* handwriting velocity (a) and its surrogate time series for participant 8. Surrogates (a), (b) and (c) are sequence-randomised, phase-randomised and Gaussian scaled versions, respectively. A lag of 5 was chosen to best display the dynamics of the system.

Table 4
The correlation dimension of the concatenated *X* velocity and concatenated *Y* velocity and their respective surrogate data sets

	n	Correlation dimension, participant 1	*n*	Correlation dimension, participant 8
X velocity	1711	3.370 ± 0.506	2364	2.851 ± 0.374
X surrogate 1	1711	6.450 ± 6.450	2364	6.443 ± 6.443
X surrogate 2	1711	4.086 ± 0.318	2364	3.775 ± 0.631
X surrogate 3	1711	4.307 ± 0.098	2364	3.975 ± 0.347
Y velocity	1714	2.886 ± 0.268	2308	2.696 ± 0.385
Y surrogate 1	1714	6.438 ± 6.438	2308	6.451 ± 6.451
Y surrogate 2	1714	3.660 ± 0.378	2308	3.887 ± 0.775
Y surrogate 3	1714	4.213 ± 0.342	2308	4.028 ± 0.268

each case there was a highly significant difference (surrogate 1, $t(18) = 126.62$, $p < 0.0001$; surrogate 2, $t(18) = 9.25$, $p < 0.0001$; surrogate 3, $t(18) = 12.66$, $p < 0.0001$).

7.5. A summary of the nonlinear dynamic analysis for eight participants

Tables 5–7 contain a summary of the results of a nonlinear dynamic analysis of handwriting for eight participants. Table 5 shows correlation

Table 5
The correlation dimension of the concatenated *X* handwriting velocity and concatenated *Y* handwriting velocity for eight adult participants

	n	*X* velocity correlation dimension	*n*	*Y* velocity correlation dimension
Participant 1	1711	3.370 ± 0.506	1714	2.886 ± 0.268
Participant 2	1840	3.322 ± 0.304	1840	2.990 ± 0.152
Participant 3	1974	3.308 ± 0.251	1974	2.879 ± 0.398
Participant 4	1454	2.828 ± 0.308	1454	2.886 ± 0.142
Participant 5	2240	3.584 ± 0.205	2238	3.444 ± 0.297
Participant 6	2000	3.008 ± 0.251	2000	3.009 ± 0.134
Participant 7	1784	3.677 ± 0.283	1784	3.130 ± 0.264
Participant 8	2364	2.851 ± 0.374	2308	2.696 ± 0.385
Average	1920	3.244	1920	2.990

Table 6
The largest Lyapunov exponent and the sum of Lyapunov exponents for *X* handwriting velocity of eight participants

X velocity	Estimated largest Lyapunov exponent, embedded in seven dimensions [a]	Sum of Lyapunov exponents
Participant 1	0.145	−1.154
Participant 2	0.119	−1.447
Participant 3	0.106	−1.557
Participant 4	0.122	−0.881
Participant 5	0.115	−1.029
Participant 6	0.106	−1.330
Participant 7	0.090	−0.731
Participant 8	0.106	−0.621
Average	0.114	−1.094

[a] The velocity time series were embedded in seven dimensions and the neural network model used seven input units, 1–14 hidden units and one output unit. The Lyapunov exponents are expressed in log base-e.

Table 7
The largest Lyapunov exponent and the sum of Lyapunov exponents for the *Y* handwriting velocity of eight participants

Y velocity	Estimated largest Lyapunov exponent, embedded in seven dimensions [a]	Sum of Lyapunov exponents
Participant 1	0.132	−1.094
Participant 2	0.130	−1.157
Participant 3	0.118	−1.138
Participant 4	0.105	−1.218
Participant 5	0.131	−0.831
Participant 6	0.111	−1.082
Participant 7	0.113	−1.060
Participant 8	0.106	−1.037
Average	0.118	−1.077

[a] The velocity time series were embedded in seven dimensions and the neural network model used seven input units, 1–14 hidden units and one output unit. The Lyapunov exponents are expressed in log base-e.

dimensions ranging between 2.7 and 3.7, suggesting that the correlation dimension of handwriting velocity profiles is small and fractional, but with substantial individual variability.

Lyapunov spectra were calculated using the program NETLE (Gençay & Dechert, 1992; Kuan & Tung, 1995). This procedure uses a multilayer feed forward neural network to generate a nonlinear model of the experimental time series that is then used to estimate the Lyapunov spectrum. By definition, the Lyapunov exponents for a dynamical system measure the average rate of divergence or convergence of a typical trajectory (Gençay & Dechert, 1992). There are n Lyapunov exponents for an n dimensional system. Using the definition of Lyapunov exponents, Gençay and Dechert state that all the Lyapunov exponents can be calculated using the Jacobian of the nonlinear function g along a trajectory $\{x_t\}$. This function is estimated by the neural network and derives from differentiating the original nonlinear mapping in the embedding space. As noted earlier, by using the method of delays the dynamics of a system can be reconstructed from its observables. The technique used in NETLE involves estimating the nonlinear function g, which relates the next time series value to its previous values based on the reconstruction and then calculating the Lyapunov exponents of g using the definition in terms of the Jacobean functions.

Multilayer feedforward neural networks can asymptotically approximate a (differentiable) function and its derivatives to any degree of accuracy and with as few as a hundred observables. For the handwriting data, since the D_2

estimate was approximately 3, the embedding dimension chosen for NETLE was 7 (i.e. $2 \times D_2 + 1$). Lyapunov exponents were calculated using seven input units with hidden units ranging from 1 to 14. The model that produced stable Lyapunov exponents was chosen and the largest Lyapunov exponent and the sum of all Lyapunov exponents was calculated. Tables 6 and 7 show that in all cases the largest Lyapunov exponents were positive and that the sum of Lyapunov exponents was negative.

8. Discussion

If handwriting involves nonlinear information processing mechanisms then it is reasonable to assume that each time a participant writes a particular word they are in fact taking a sample from the attractor for that movement. This proposition breaks down if errors or uncharacteristic pauses disturb the handwriting trace during a long writing period. It was hypothesized that the dynamic analysis results achieved for the concatenated time series would be equivalent to those obtained for the time series for each sample, with the advantage of producing more accurate results. In this study support for this proposition was obtained by examining five temporally disjointed samples of the known attractors. These samples were created by using a 'noise' time series and two known chaotic time series and removing short chunks from the data every few hundred data points.

As hypothesized, qualitatively in terms of the delay portraits and quantitatively in terms of correlation dimension, the chunked time series displayed compatible characteristics to the original time series. This means that by concatenating five samples of the same attractor we are not substantially altering the time series dynamics. It is particularly noteworthy that the structure in structured time series is not destroyed and conversely no structure is introduced into noisy, unstructured time series.

When the dynamic characteristics of the five handwriting trials and the five concatenated trials were examined, each of the five trials was very similar to each other both qualitatively in terms of the delay portraits and quantitatively in terms of the correlation dimension. The trials displayed compatible characteristics to those evident when the five trials were concatenated. This was true for both the X and Y handwriting velocity profiles for the two participants whose data were analyzed in detail.

When we are dealing with a stable attractor, these results confirm the validity of the pre-processing technique of concatenation of trials to form larger data sets. For handwriting velocity attractors, concatenation of five trials sampled at 100 Hz provides time series long enough for the calculation of stable and reliable results. Furthermore, since relatively stable results could be obtained for the dynamic analysis, SVD appears to have reduced the noisy random variation within the data to an acceptable level. This provides us with a useful technique for nonlinear analysis of short multiple samples of handwriting attractors, so obviating a requirement for long time series with minimum noise in order to obtain accurate results.

We argued that the results of a dynamical analysis of the concatenated time series would be substantially different to those gained from an analysis of surrogate data sets based on the original data. In particular, it was hypothesized that the phase portraits for the surrogate time series would display less structure and that the correlation dimensions would be significantly larger. This hypothesis was supported by the results for both the X and Y handwriting velocity profiles for the two participants. The concatenated time series were found to be substantially dissimilar to the surrogate time series and therefore different from uncorrelated noise, linearly autocorrelated noise as well as a static linear transform of Gaussian noise.

The use of SVD on data which has been calculated by taking the first difference of a position time series could be criticized since this method implies a strong correlation at lag 1. It can perhaps be argued that this may explain the differences between the original time series and the uncorrelated surrogates (i.e. the sequence-randomized surrogate). However it does not explain the differences between the original and the two surrogates which maintain the original autocorrelation structure (i.e. the phase-randomized surrogate and the generalization of this to non-Gaussian noise). This point emphasizes the need to use several different surrogates to confirm the validity of the nonlinear dynamic analysis.

As was hypothesized, the correlation dimensions of the X and Y handwriting velocity time series for each of the eight experimental participants were less than five and fractional. This tells us that the attractor generating the handwriting studied in this experiment appears to be low dimensional. This finding was robust, similar results being calculated for all eight participants, for both X and Y velocity time series, using several techniques as well as various delay parameters. The hypothesis that the largest Lyapunov exponent for each concatenated time series would be positive while the sum of the Lyapunov exponents was negative was also supported by the results. While these findings need to be treated with some caution, the discovery that reliable, consistent and stable results were established for each participant for individual trials as well as for the concatenated time series argues strongly for their validity. This is further supported by the finding that these results were substantially different from those found for the surrogate data sets. Our analysis provides strong evidence for the proposal that handwriting dynamics are chaotic.

The low fractional correlation dimension found for handwriting velocity profiles supports the findings of Dooijes and Struzik (1994). These researchers calculated the handwriting dimensionality using both the static output as well as the dynamics. While commenting on the inherent difficulties in calculating precise dimensional estimates, they concluded that handwriting has a fractal dimension approximately between 1 and 2. The present study found fractional correlation dimensions between 2 and 4. It is possible that the techniques used by Dooijes and Struzik underestimated the fractal dimension for handwriting since they analyzed static data either as a series of co-ordinates, or simply as a word on a page. In the current study we examined dynamic velocity data so it is reasonable to expect a more complex (i.e. higher dimension) time series.

These results support the findings of Akamatsu et al. (1986) who found that a muscle model based on the classic length tension curves could produce an inherent chaotic oscillation during contraction. The particular model developed by these researchers displayed a largest Lyapunov exponent of 0.157 log base-e (i.e. 0.227 bits per iteration), while in the present study the average largest Lyapunov exponent was found to be 0.114 and 0.118 log base-e for the X and Y velocities, respectively. While these results differ slightly, it is worth noting that the largest Lyapunov exponent for the muscle model depends on changes in the parameter related to muscle activation. Since Akamatsu et al. used an arbitrary value of this parameter so that the model displayed an example of chaotic characteristics, perhaps a slight change in this parameter could lead to the largest Lyapunov exponents of the order found for handwriting velocity profiles. In any case, there is no guarantee that the scaling of Lyapunov exponents should be similar for these rather different motor time series.

These results are consistent with those found by Kay (1988) who concluded that simple rhythmic finger movements are low dimensional. Finally, the present findings support those of Mitra et al. (1997, 1998) who found simple rhythmic motor movements can be characterized as chaotic evolutions on a strange attractor.

The finding that handwriting velocity profiles are chaotic, if confirmed, has important implications for theories of skilled performance. A recent comprehensive theory of human performance supposes that the psychomotor system contains background neuromotor noise so that the attainment of skilled performance is partly due to noise minimization (van Galen et al., 1993; van Gemmert & van Galen, 1997). Van Gemmert and van Galen (1997) propose initially that the psychomotor system is noisy. Part of this neuromotor noise originates centrally resulting in interference with concurrent tasks, distractions and effects of task complexity. Within the biomechanical system it is thought that neuromotor noise results from spontaneous neuromotor tremors and motor recruitment noise.

The theory also proposes that noise propagates within the information processing system on a time- and space-related basis, and that a noisy system does not necessarily imply performance deterioration. Noise activates and alerts the system's processing capability, however as the noise level increases it decreases the signal to noise ratio leading to an increase in errors. This result is similar to the classic Yerkes–Dodson law for human performance under stress. A key difference between this theory and the neuromotor noise theory is the finding that the psychomotor system can modulate parameters related to muscle stiffness and friction to alter the signal to noise ratio. When these parameters increase, there is a compensatory increase in movement speed that leads to more noise. After an optimal level of movement speed, errors rapidly increase to a point where accurate performance is impossible. According to this theory, accurate skilled performance requires the psychomotor system to operate with an optimum signal-to-noise ratio.

In neuromotor noise theory, the noise is thought of as random background variation produced in part by physiological tremor within the muscles. However, the present study, along with other recent work on movement dynamics, postulates that the neuromotor noise, rather than being random variation, is in fact the output of a chaotic tremor process (Babloyantz & Lorenço, 1994; Mitra et al., 1997, 1998). If this is true then it implies that the task of the motor system is not simply to minimize random variation, but to minimize chaotic variation. While not altering the basic neuromotor noise theory, when stated in these terms the task of the psychomotor system could be thought of as an exercise in the reduction of dynamical degrees of freedom to produce movements that are optimally stable.

This paper has demonstrated that each handwriting trial is a sample of the same strange attractor. This attractor appears characteristic for a participant writing a particular word and so may reflect a unique dynamic motor memory for that movement. If the handwriting dynamics are chaotic, then we may have solved the degrees of freedom problem. A chaotic oscillator can reduce the seemingly infinite degrees of freedom to the few actually observed when a movement is produced. With slight variations of the strange attractor's few parameters, an infinite variety of desired movements could be produced. This variety of movement results in a consistent product that has been generated in a large number of different environmental contexts. While this theory is still speculative, it is consistent with the evidence presented above.

The goal of further research will be to identify the exact source of the chaos. It could merely be an uncontrollable artifact of the biomechanical system, or reflect some important aspect of the cognitive system. Future work will be aimed at confirming that the chaotic dynamics form an important aspect of motor memory.

Our next stage of research will be aimed at using chaos analysis techniques to examine individual differences between healthy participants and those who suffer motor skill degradation due to multiple sclerosis. It is hoped that characteristic differences will be found that can be used in a clinical setting for the early detection of this disease, as well as for monitoring the treatment of this disease.

This paper has demonstrated the validity and reliability of concatenation of handwriting velocity time series as well as the use of SVD to provide long time series with minimum noise. It has confirmed the utility of delay portraits in the analysis of dynamical systems as well as nonlinear dynamic analysis as a novel approach for the analysis of handwriting. We have provided strong evidence for the proposal that a chaotic process generates handwriting dynamics. Finally we have speculated about possible implications this finding has for theories of motor memory. It is our belief that for handwriting, the motor production involves a chaotic oscillator whose parameters change due to the given environmental and biomechanical context. This motor memory consists of the boundary conditions of a functionally specific coordinate structure that allows the chaotic attractor for handwriting to unfold. Due to its chaotic nature, this attractor is able to generate the infinite variety of movements found in this complex psychomotor skill.

Acknowledgements

This research was supported by a Research Grant from the Australian Research Council awarded to R. Heath and A. Heathcote. Mitchell Longstaff's research was supported by a University of Newcastle Postgraduate

Award. The authors would like to thank the two anonymous reviewers whose useful comments helped to improve this paper.

References

Akamatsu, N., Hannaford, B., & Stark, L. (1986). An intrinsic mechanism for the oscillatory contraction of muscle. *Biological Cybernetics*, *53*, 219–227.

Armstrong, C. F., Huxley, A. F., & Julian, F. J. (1966). Oscillatory responses in frog skeletal muscle fibres. *Journal of Physiology*, *186*, 26–27.

Aubry, N., Holmes, P., & Lumley, J. L. (1988). The dynamics of coherent structures in the wall region of a turbulent boundary layer. *Journal of Fluid Mechanics*, *192*, 115–173.

Babloyantz, A. (1985). Strange attractors in the dynamics of brain activity. In H. Haken, *Complex systems – operational approaches in neurobiology, physics, and computers* (pp. 116–122). Berlin: Springer.

Babloyantz, A. (1991). Evidence for slow brain waves: A dynamical approach. *Electroencephalography & Clinical Neurophysiology*, *78*, 402–405.

Babloyantz, A., & Destexhe, A. (1986). Low-dimensional chaos in an instance of epilepsy. *Proceedings of the National Academy of Science USA*, *83*, 3513–3517.

Babloyantz, A., & Lourenço, C. (1994). Computation with chaos: A paradigm for cortical activity. *Proceedings of the National Academy of Science USA*, *91*, 9027–9031.

Barlow, J. S., Creutzfeldt, O. D., Michael, D., Houchin, J., & Epelbaum, H. (1981). Automatic adaptive segmentation of clinical EEGs. *Electroencephalography and Clinical Neurophysiology*, *51*, 512–525.

Bernstein, N. (1967). *The Coordination and Regulation of Movement*. Oxford: Pergamon Press.

Dooijes, E. H. (1983). Analysis of handwriting movements. *Acta Psychologica*, *54*, 99–114.

Dooijes, E. H., & Struzik, Z. R. (1994). The fractal dimension of handwriting. In: *Proceedings of the IEEE electronics division European workshop on handwriting analysis and recognition: A European perspective* (pp. 1–8). Brussels. London: IEE Press.

Fox, J. R., & Randell, J. E. (1970). Relationship between forearm tremor and the biceps electromyogram. *Journal of Applied Physiology*, *29*, 103–108.

Gençay, R., & Dechert, W. D. (1992). An algorithm for the *n* Lyapunov exponents of an *n*-dimensional unknown dynamical system. *Physica D*, *59*, 142–157.

Gottman, J. M. (1981). *Time series analysis: A comprehensive introduction for social scientists*. Cambridge: Cambridge University Press.

Grassberger, P., & Procaccia, I. (1983). Characterization of strange attractors. *Physical Review Letters*, *50*, 346–349.

Hollerbach, J. M. (1981). An oscillation theory of handwriting. *Biological Cybernetics*, *39*, 139–156.

Joyce, G. C., & Rack, P. M. H. (1974). The effects of load and force on tremor at the normal human elbow joint. *Journal of Physiology*, *240*, 375–396.

Kay, B. (1988). The dimensionality of movement trajectories and the degrees of freedom problem: A tutorial. *Human Movement Science*, *7*, 343–364.

Kuan, C. -M., & Tung, L. (1995). Forecasting exchange rates using feedforward and recurrent networks. *Journal of Applied Econometrics*, *10*, 345–364.

Kugler, P. N., & Turvey, M. T. (1987). *Information natural, law, and the self-assembly of rhythmic movement*. Hillsdale, NJ: Erlbaum.

Kugler, P. N., Kelso, J. A. S., & Turvey, M. T. (1980). On the concept of coordinative structures as dissipative structures: I. Theoretical lines of convergence. In G. E. Stelmach, & J. Requin, *Tutorials in motor behavior* (pp. 3–47). Amsterdam: North-Holland.

Longstaff, M. G., & Heath, R. A. (1997). Space-time invariance in adult handwriting. *Acta Psychologica*, *97*, 201–214.

Lumley, J. L. (1970). *Stochastic tools in turbulence*. New York: Academic Press.

Luttgens, K. L., & Wells, K. F. (1982). *Kinesiology: Scientific basis of human motion*. Philadelphia, PA: Saunders College.

Maarse, F. J., van Galen, G. P., & Thomassen, A. J. W. M. (1989). Models for the generation of writing units in handwriting under variation of size slant and orientation. *Human Movement Science*, *8*, 271–288.

Mayer-Kress, G., Yates, F. E., Benton, L., Keidel, M., Tirsh, W., Popl, S. J., & Geist, K. (1988). Dimensional analysis of non-linear oscillations in brain heart and muscle. *Mathematical Biosciences*, *90*, 155–182.

Mitra, S., Amazeen, P. G., & Turvey, M. T. (1998). Intermediate motor learning as decreasing active (dynamical) degrees of freedom. *Human Movement Science*, *17*, 17–65.

Mitra, S., Riley, M. A., & Turvey, M. T. (1997). Chaos in rhythmic movement. *Journal of Motor Behavior*, *29*, 195–198.

Nerenberg, M. A. H., & Essex, C. (1990). Correlation dimension and systematic geometric effects. *Physical Review A*, *42*, 7065–7074.

Ott, E. (1993). *Chaos in dynamical systems*. Cambridge, UK: Cambridge University Press.

Packard, N. H., Crutchfield, J. P., Farmer, J. D., & Shaw, R. S. (1980). Geometry from a time series. *Physical Review Letters*, *45*, 712–716.

Plamondon, R., & Clement, B. (1991). Dependence of peripheral and central parameters describing handwriting generation on movement direction. *Human Movement Science*, *10*, 193–221.

Rapp, P. E. (1994). A guide to dynamical analysis. *Integrative Physiological and Behavioural Science*, *29*, 311–323.

Rosenstein, M. T., Collins, J. J., & De Luca, C. J. (1993). A practical method for calculating largest Lyapunov exponents from small data sets. *Physica D*, *65*, 117–134.

Rosenstein, M. T., Collins, J. J., & De Luca, C. J. (1994). Reconstruction expansion as a geometry-based framework for choosing proper delay times. *Physica D*, *73*, 82–98.

Rowlands, G., & Sprott, J. C. (1992). Extraction of dynamical equations from chaotic data. *Physica D*, *58*, 251–259.

Ruelle D. (1981). Chemical kinetics and differentiable dynamical systems. In A. Pacault, & C. Vidal, *Nonlinear Phenomena in chemical dynamics*, Berlin: Springer.

Saltzman, E. L., & Kelso, J. A. S. (1987). Skilled actions: A task dynamic approach. *Psychological Review*, *94*, 84–106.

Sauer, T. (1992). A noise reduction method for signals from nonlinear systems. *Physica D*, *58*, 193–201.

Sheridan, M. R. (1984). Response programming response production and fractionated reaction time. *Psychological Research*, *46*, 33–47.

Shuster, H. G. (1988). *Deterministic chaos: An introduction*. Weinheim: VCH.

Skinner, J. E., Molnar, M., & Tomberg, C. (1994). The point correlation dimension: Performance with nonstationary surrogate data and noise. *Integrative Physiological and Behavioral Science*, *29*, 217–234.

Skinner, J. E., Carpeggiani, C., Landisman, C. E., & Fulton, K. W. (1991). The correlation-dimension of the heartbeat is reduced by myocardial ischemia in conscious pigs. *Circulation Research*, *68*, 966–976.

Skinner, J. E., Goldberger, A. L., Mayer-Kress, G., & Ideker, R. E. (1990). Chaos in the heart: Implications for clinical cardiology. *Biotechnology*, *8*, 1018–1924.

Skinner, J. E., Martin J. L., Landisman, C. E., Mommer, M. M., Fulton, K., Mitra, M., Burton, W. D., & Saltzberg, B. (1990). Chaotic attractors in a model of neocortex: Dimensionalities of olfactory bulb surface potentials are spatially uniform and event related. In E. Basar, *Chaos in brain function*, 119–134, Berlin: Springer.

Sprott, J. C., & Rowlands, G. (1995). *Chaos data analyzer: The professional version*. New York: American Institute of Physics.

Takens, F. (1981). Detecting strange attractors in turbulence. *Lecture Notes in Mathematics*, *898*, 366–381.

Theiler, J., Eubank, S., Longtin, A., Galdrikian, B., & Farmer, J. D. (1992). Testing for nonlinearity in time series: The method of surrogate data. *Physica D*, *58*, 77–94.

Tsonis, A. A. (1992). *Chaos: From theory to applications*. New York: Plenum Press.

van Galen, G. P., van Doorn, R. R. A., & Schomaker, L. R. B. (1990). Effects of motor programming on the power spectral density function of finger and wrist movements. *Journal of Experimental Psychology: Human Perception and Performance*, *16*, 755–765.

van Galen, G. P., Portier, S. J., Smits-Engelsman, B. C. M., & Schomaker, L. R. B. (1993). Neuromotor noise and poor handwriting in children. *Acta Psychologica*, *82*, 161–178.

van Gemmert, A. W. A., & van Galen, G. P. (1997). Stress, neuromotor noise and human performance: A theoretical perspective. *Journal of Experimental Psychology: Human Perception and Performance*, *23*, 1299–1313.

Wann, J., & Nimmo-Smith, I. (1991). The control of pen pressure in handwriting: A subtle point. *Human Movement Science*, *10*, 223–246.

Wolf, A., Swift, J. B., Swinney, H. L., & Vastano, J. A. (1985). Determining Lyapunov exponents from a time series. *Physica D*, *16*, 285–317.