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Trajectory Formation and Handwriting: A Computational Model

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Abstract. This paper proposes a computational model for different aspects of trajectory formation, from point-to-point movements to handwriting. The proposed model is based on a mechanism of composition of basic curve elements (strokes) which separates the spatial and the temporal aspects of trajectory formation. At the same time, the model suggests a method for storing and describing arm movements, as a list of stroke descriptors. Experimental trajectories were digitized and analyzed with regard to several types of movements: i) point-to-point trajectories, ii) closed trajectories, iii) trajectories with inflection points, iv) spiral-like trajectories, v) handwritten trajectories. Velocity and curvature profiles were computed for the trajectories and the model was fitted to the data. The implications of the model and its "credibility" in the general context of motor control are discussed.

1. Introduction

Trajectory Formation is one of the basic functions of the neuromotor controller, such as the compensation of loads, the pursuit of moving targets, the appropriate control of impacts (e.g. hitting), and the generation of contact forces (e.g. pushing).

Motor skills in real life involve basic motor functions either in simultaneous combination or in complex sequences, but in this paper we focus on those aspects of motor behaviour where "pure" trajectory formation can be assumed to be predominant or, at least, can be studied separately from other important aspects, such as the motor-cognitive aspect.

In particular, reaching, pointing, avoiding, generating scribbles, drawing, handwriting and gesturing are different motion paradigms which all result in the generation of planar or spatial trajectories of different degrees of complexity.

A possible hypothesis, which will be discussed in this paper, is that different paradigms of trajectory formation share a common generation mechanism. This hypothesis is supported by the fact that similar kinematic patterns can be observed for simple movements (e.g. reaching) and for complex movements as well (e.g. handwriting). In particular, we refer to the strong coupling which can be observed between the time course of the velocity of the hand and the curvature of the trajectory (Teulings and Thomassen, 1979; Viviani and Terzuolo, 1980; Abend et al., 1982).

Nevertheless, it is worth noting that most studies on handwriting have emphasized the closeness of this motor activity with other motor activities (like speech and typewriting) which share with handwriting a cognitive aspect and not an aspect of trajectory formation (Mermelstein and Eden, 1964; Eden, 1968; van Galen, 1980).

We now formulate what we regard the key question: which is the common mechanism of trajectory formation?

Two classes of models of this mechanism can be distinguished: space-oriented and muscle-oriented models.

For a muscle-oriented model, the mechanism of trajectory formation is directly related to the geometry of the muscles and to their mechanical properties. For example, the models proposed by Denier Van Der Gon and Thuring (1965), Eden (1968), Vredenbregt and Koster (1971), and Hollerbach (1981) rely on two orthogonal groups of muscles and on their elastic properties for explaining the trajectories of the hand during handwriting. In addition to these models, which represent anatomy in a rather simplified way, an attempt to take into account in more detail what is known from muscle geometry was done by Hogan (1981), who explored the role of double joint muscles in

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generating approximately straight trajectories in various parts of the work field, for two-joint planar movements.

For a space-oriented model, the trajectory formation mechanism is based on the capability of expressing and controlling the trajectory of the hand in space, independently of the actual joint and muscle patterns. This hypothesis can be made simply on logical grounds (Lashley, 1951; Bernstein, 1967), since it is desiderable for the arm control system to assure a behaviour of the hand which is independent of the particular configuration of the joints, but it is also supported by the well known fact that people can write and draw in the same way on a sheet of paper and on a blackboard (Katz, 1951), using the hand or even other parts of the body. Furthermore, it was found that the kinematic patterns of the hand are invariant with respect to variations of the starting point, of inclination and of size (Viviani and Terzuolo, 1980; Morasso, 1981; Abend et al., 1982).

Muscle oriented models are logically two level models, in which we can distinguish a cognitive/symbolic level and a joint/muscular level. They assume the existence of a mechanism of coding/planning and a mechanism of actuation. While the former mechanism implies information processing and decision making, the latter mechanism performs geometric/mechanical transformations in an implicit, "analogic" way (van Galen, 1980).

Space oriented models, in addition to the two previous levels, also assume an intermediate level which generates spatio-temporal patterns for a given planned movement. As a consequence, these models require a mechanism for transforming spatial sequences into angular/muscular sequences: such mechanism implies information processing [i.e. the solution of the so called "inverse kinematic problem", Benati et al., (1980, 1982)] but not decision making and it is ideally suited for an implementation as a dedicated, analog computing network. The model presented in this paper is space-oriented and is mainly concerned with the first two levels.

Another important characteristic of trajectory formation is the "discrete" nature of complex trajectories. This is evident in handwriting, which is usually interpreted as a chain of "strokes", particularly in pattern recognition studies (Mermelstein and Eden, 1964; Crane, 1977; Herbst and Liu, 1977) which are based on techniques of segmentation of the global trajectory and on a characterization of each segment.

Two different hypothesis can be formulated about strokes:

i) stokes are actual segments of the trajectory, i.e. they are immediately observable from the recorded movement;

ii) strokes are "hidden", i.e. the actual trajectory results from the composition of overlapped segments (Morasso and Mussa Ivaldi, 1981).

Existing models of handwriting agree with the former hypothesis, whereas the present paper supports the latter. The rationale of this apparently more complicated definition of stroke is that it allows to separate the two basic elements of any mechanism of composite generation of trajectories: i) chaining of component segments and ii) smooth transition between consecutive segments.

Summing up, the model of trajectory formation discussed in the next section is a space-oriented model which uses a mechanism of composition of discrete strokes, capable to generate smooth trajectories by means of the simple time overlap of different strokes. At the same time, the model suggests a coding/storage of the trajectory as a discrete list of stroke descriptors.

2. A Model of Trajectory Formation

The model of trajectory formation proposed in this paper is defined and implemented by using and generalizing typical methods of computational geometry and computer graphics, i.e. techniques for generating Parametric Composite Curves (de Boor, 1978; Faux and Pratt, 1979).

In general, composite curves are obtained by linking segments of properly chosen base curves (strokes, in our terminology). The linking or jointing technique must be able to preserve certain continuity conditions, in order to generate sufficiently smooth trajectories.

The class of composite trajectories we are concerned about is characterized by the fact of having a continuous velocity and a continuous curvature, because man made planar trajectories display this feature. As a consequence, each stroke (expressed in parametric terms, using time as the running parameter) must be represented by at least a cubic polynomial function of time and the jointing technique between strokes must guarantee continuity up to the second time derivative[1].

Let us now focus on the mechanism of composition among consecutive strokes. Such mechanism must be able, on one hand, to shape the generated curve according to a global geometric description (given, for example, as a set of points in the plane) and, on the other hand, it must be able to guarantee local geometric characteristics, such as a sufficient smoothness.

1 For spatial trajectories, in addition to velocity and curvature, it is also necessary to consider torsion, which involves the third time derivative. Spatial piecewise cubic curves have torsion discontinuities. However, we are not aware of experimental data which show whether or not man made trajectories have continuous torsion

Some methods used in computer graphics provide rules for satisfying the two conditions simultaneously (e.g. composite Bezier curves and regular spline functions). The resulting trajectory consists then of patches of polynomial curves (the strokes), which are immediately detectable.

Alternatively, it is possible to separate the problems of shaping and of smoothing by providing a set of base functions with built-in smoothness characteristics. This is the case, for example, of B-splines, which play (in the geometry of curves) a similar role to the role played by sinusoidal waveforms in signal processing. The generated trajectory, in this case, results from a weighted sum of B-splines, which are overlapped in time (in a similar way to the representation of a signal as a sum of modulated and phase delayed sinusoidal waveforms). As a consequence, if we associate the notion of stroke with the curve segment generated by each B-spline, it follows that strokes are not immediately observable from the generated trajectory but are "hidden".

In the field of computer graphics, the two types of composition mechanisms are completely equivalent. In the field of motor control, the latter mechanism is much more attractive because it allows to devide the generation system into two components: i) a component which is concerned only with the geometric characterization of the movement (e.g. a polygonal curve which outlines the planned trajectory), and ii) one or more function generators appropriately timed.

Rectilinear Strokes

Let us define a "rectilinear stroke" as a planar curve in parametric form (with time as the running parameter) which is generated according to the following rule:

$$\underline{r}=\underline{r}(t)=\underline{l}\,\underline{\underline{R}}\,\underline{s}_r(t)\,, \tag{2.1}$$

where $\underline{l}$ is the length of the stroke, $\underline{\underline{R}}$ is a 2×2 rotation matrix, which identifies the tilt of the stroke, and $\underline{s}_r(t)$ is a normalized stroke of unit length, directed for example along the x-axis:

$$\underline{s}_r(t)=u(t;T)\underline{i} \tag{2.2}$$

$u(t;T)$ is the time generating function, i.e. a step-like function of time which goes from 0 to 1 in T seconds with a bell-shaped velocity profile and with a continuous second time derivative.

Composite trajectories are generated according to the following rule:

$$\underline{r}=\underline{r}(t)=\underline{r}_1(t-\tau_1)+\underline{r}_2(t-\tau_2)+\ldots+\underline{r}_n(t-\tau_n) \tag{2.3}$$

which is characterized by i) a time overlap pattern of a sequence of strokes, and ii) a linear superposition of the delayed strokes.

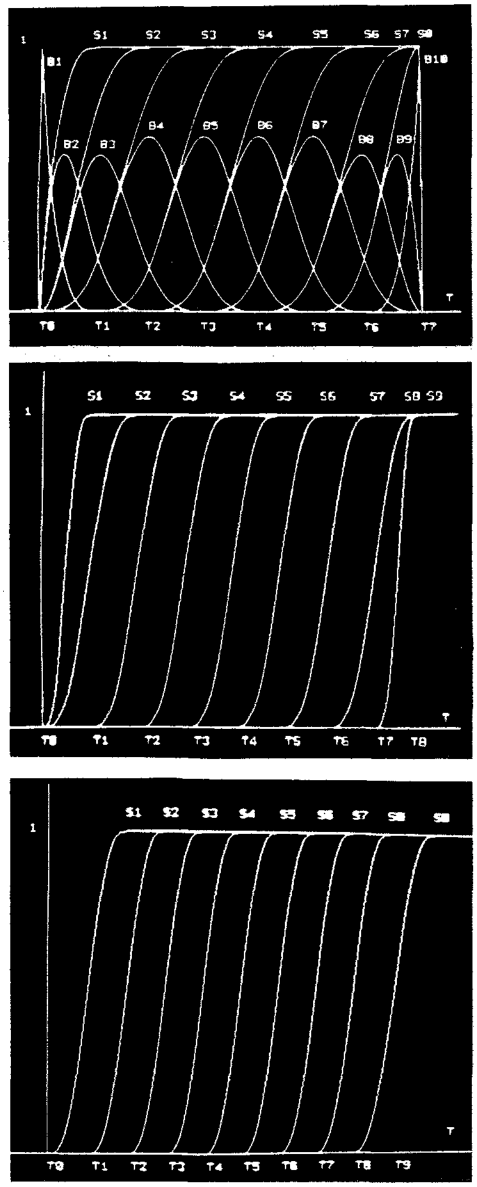

Fig. 1. Overlapping patterns of step-shaped stroke generating functions. Bottom: the generating functions are delayed in such a way that two of them are simultaneously active, except for the first half function and the last half function. Middle: same as the previous one, also at the two borders. Top: the generating functions are delayed in such a way that three of them are active simultaneously. In this case, the stroke generating functions become S-splines (Appendix A) and the figure shows also the corresponding bell-shaped B-splines

If the time delays are chosen in such a way to have no overlap between consecutive strokes, the composition rule (2.3) generates a polygonal curve of straight segments. As the overlap increases, smoother and smoother curves will result. In particular, it is possible to show that, if the time delays are chosen in such a way that three strokes are active for each time

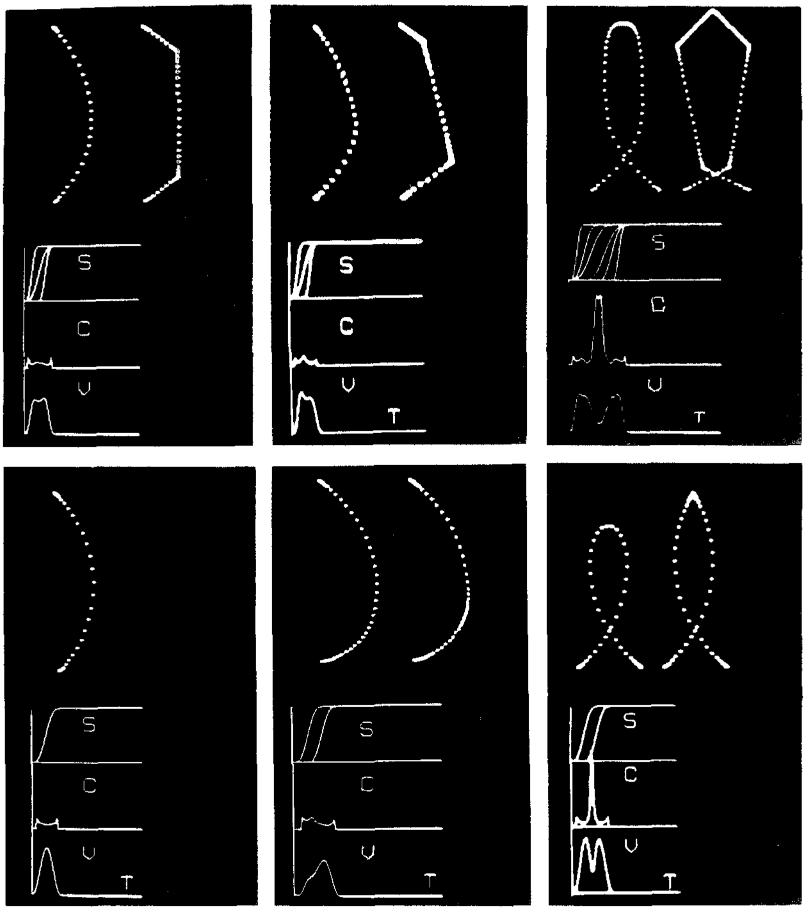

Fig. 2. Trajectories generated by the stroke composition model. Upper row: straight strokes with triple overlapping. Lower row: circular strokes with double overlapping. *S*: sequence of stroke generating functions. *V*: Velocity profile. *C*: curvature profile. For each frame, the upper-left part shows the generated trajectory, and the upper-right part shows the underlying chain of strokes

instant and if the time generating functions are cubic polynomial functions of time, the mechanism generates cubic spline curves (Appendix A).

Circular Strokes

The same idea can be generalized using curved segments as strokes. For example, "circular strokes" can be defined as follows:

$$\underline{r}=\underline{r}(t)=\underline{l}\underline{R}\underline{s}_c(t)\,, \tag{2.4}$$

where $\underline{l}$, $\underline{R}$ have the same meaning as before and $\underline{s}_c(t)$ (the normalized circular stroke) is given by

$$\underline{s}_c(t)=(1/2\sin a/2)\left((\sin(a(u(t;T)-1/2))+\sin a/2)\underline{i} + (-\cos(a(u(t;T)-1/2))+\cos a/2)\underline{j}\right) \tag{2.5}$$

which generates a segment of circle with a unitary cord, with a subtended angle "a" (the "total angular change"), and with the same velocity profile as (2.2). "a" measures the "curvedness" of the stroke and it is related to the radius of curvature p by $\underline{l}=2p\sin a$.

Composite trajectories can be generated using the same rule (2.3); the underlying polygonal curve, in this case, has curved segments.

Figure 1 shows three different types of time-overlap pattern of the time genarating functions and Fig. 2 shows some example of composite curve generation with straight and circular strokes, respectively.

Composite trajectories generated by straight strokes with triple time overlapping and by circular strokes with double time overlapping have locally the same number of degrees of freedom (d.o.f.), because straight strokes have 2 d.o.f. (length and tilt) whereas circular strokes have 3 d.o.f. (lenght, tilt, and total angular change). Both mechanisms generate trajectories with smooth velocity and curvature profiles.

The polygonal curve for circular strokes with double overlapping is closer to the actual trajectory than the corresponding polygonal curve for straight strokes with triple overlapping and, in particular, it is tangent to it at the midpoint of each stroke. We also found that the former mechanism gives a better fit to the experimental data on trajectory formation (i.e. trajectory, velocity, and curvature profiles) and therefore we chose a model of trajectory formation which uses curved strokes with double time overlapping.

The model can be represented by the block diagram of Fig. 3, where we distinguish:

i) a phase of spatial planning, in which the geometric parameters of the chain of strokes are coded and stored in a short-term motor memory,

ii) a phase of spatio-temporal stroke generation, where two identical stroke generators are active (with proper relative timing),

iii) a phase of stroke superposition,

iv) a phase of coordinate transformation (i.e. space to joint mapping) which generates the joint law of motion.

An important characterization of the mechanism is that a complete determination of the trajectory is not necessary before execution, since for each time instant the trajectory is under the influence of the two currently active strokes only. As a consequence, the process of spatial planning which feeds stroke parameters in the short term memory can evolve asynchronously with respect to the stroke generators and, in particular, it can back-track and re-program in case of emergencies or actions contingent to environment conditions.

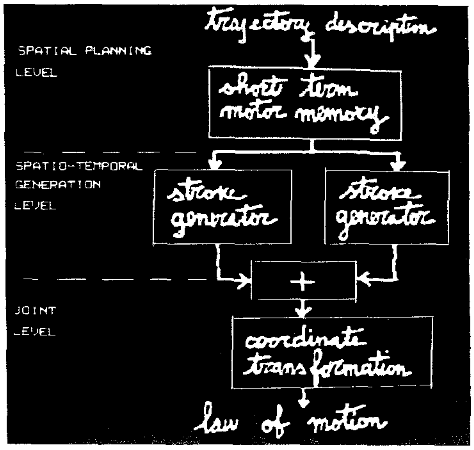

Fig. 3. Block diagram of a stroke composition model of trajectory formation

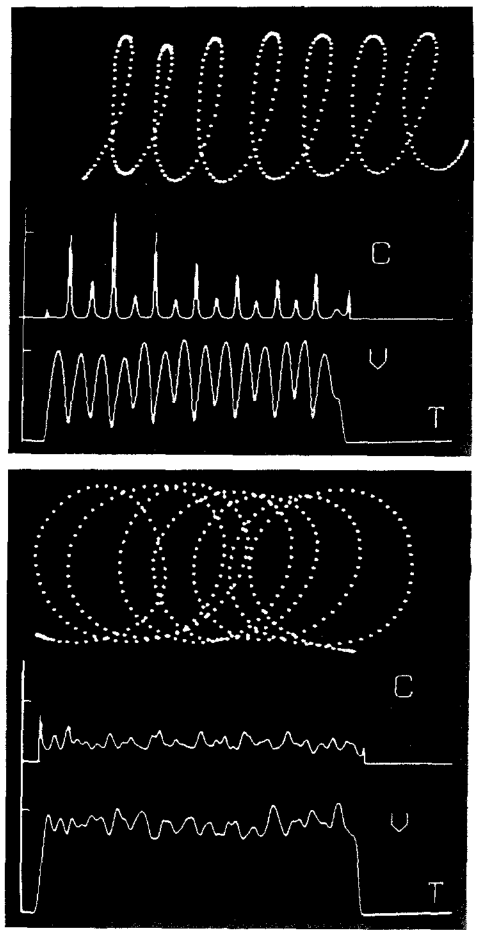

Fig. 4. Sequence of 1-shaped trajectories (top) and sequence of 0-shaped trajectories (bottom). C: curvature (calibration 0.2 mm^{-1}). V: velocity (calibration 600 mm/s). T: time (calibration 10 s)

3. Comparing the Model with Experimental Data

Different types of human arm trajectories of the hand were digitized and analyzed: point-to-point trajectories, closed trajectories, trajectories with inflection points, spiral trajectories, handwritten trajectories.

The data were collected by means of a Summagraphic digitizing tablet, at a rate of 50 samples/sec and with a resolution of 0.1 mm. Velocity of the x and y components of the recorded trajectories were estimated by approximating the data with moving-window least squares polynomials and by calculating the derivatives of such polynomials. Second time derivatives were obtained by using the procedure twice. The tangential velocity of the hand v and the curvature of the hand trajectory c (with a positive sign for counterclockwise rotations) were computed by the two formulas:

$$v=(\dot{x}^2+\dot{y}^2)^{1/2},$$

$$c=(\dot{x}\ddot{y}-\dot{y}\ddot{x})/v^3.$$

The data were fitted to the model of the previous section, taking into account the trajectory, the velocity profile and the curvature profile (the fitting procedure is discussed in Appendix B).

Five subjects partecipated in the experiments, in which they performed hand movements in response to a verbal command. No significant difference was observed whether their eyes were open or not.

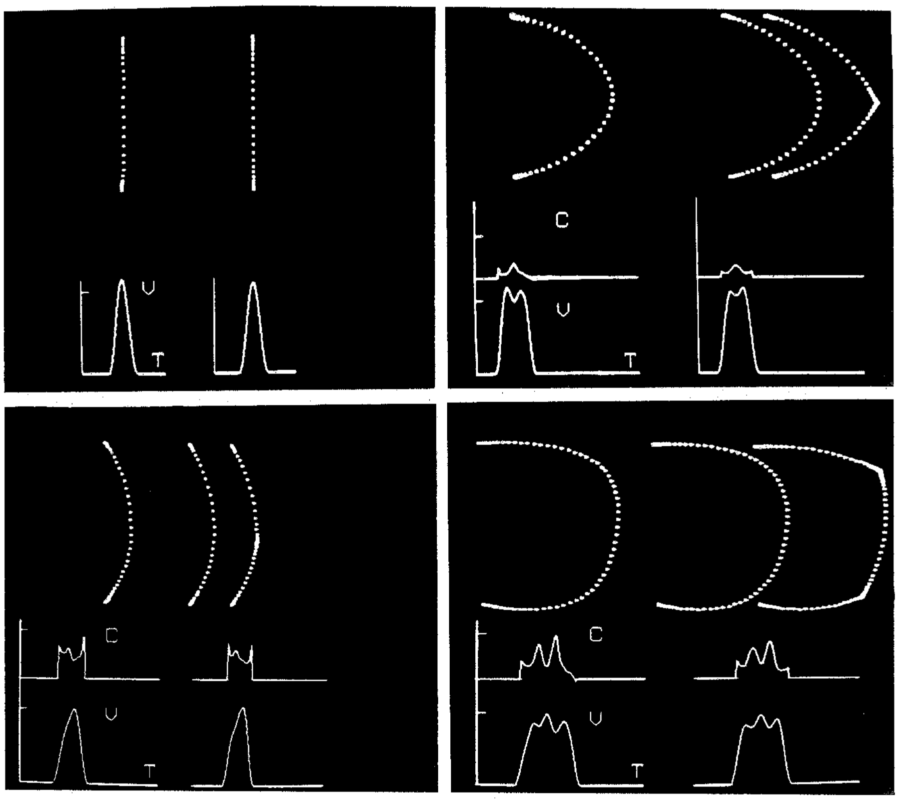

Fig. 5. Point-to-point trajectories. For each frame, the left part shows the experimental trajectory, together with the velocity profile (V) and the curvature profile (C); the right part shows the model-generated trajectory and the underlying chain of strokes. Velocity calibration: 500 mm/s. Curvature calibration: 3×10^{-2} mm^{-1} (top-right), 8×10^{-3} mm^{-1} (bottom). Time calibration: 2 s (top-left), 4 s (right), 3.5 s (bottom-left)

3.1. Point-to-Point Trajectories

In this experiment, the subjects were asked to move their hand between two specified points either directly or by means of smooth curved trajectories, ranging from shallow to highly curved ones. As already reported by Abend et al. (1981), it is possible to observe that

i) straight movements exhibit highly stereotyped characteristics, particularly a symmetric, bell-shaped velocity profile;

ii) curved movements are more irregular, particularly movements with a very small curvature: sometimes they exhibit two or more well distinct velocity peaks, sometimes the velocity profile is single-peaked but asymmetric, sometimes it is single-peaked but with a rather flat dome;

iii) on average, curved movements (even for the same length) have a significantly longer duration.

In qualitative terms, these facts agree with the general characteristics of our model of trajectory formation, discussed Appendix B. In particular, the longer duration of curved movements is consistent with the hypothesis of overlapped strokes (if we admit that straight movements are single-stroke movements and that an increased curvature is likely to require a greater number of strokes) and the irregularly of the shallow curvature movements is consistent with the sensitivity of the generation mechanism to small parameter changes when consecutive strokes are nearly parallel (if small curvature movements are multistroke, then the composing strokes are likely to be nearly parallel).

The "irregularity" which derives from near-parallelism is illustrated in Fig. 4, which compares the velocity/curvature profiles of a sequence of circular shapes and of elliptical shapes: paradoxically, the velocity/curvature pattern in the former case is more irregular then in the latter (the paradox is due to the fact that, according to common sense, a circle is usually considered "more regular" than an ellypse).

Figure 5 shows the result of fitting the model to straight or curved point-to-point movements which show some of the different patterns previously described.

3.2. Closed Trajectories

If a person is asked to draw closed trajectories of different degrees of "roundness", the recorded data have a velocity profile which may exhibit two or more

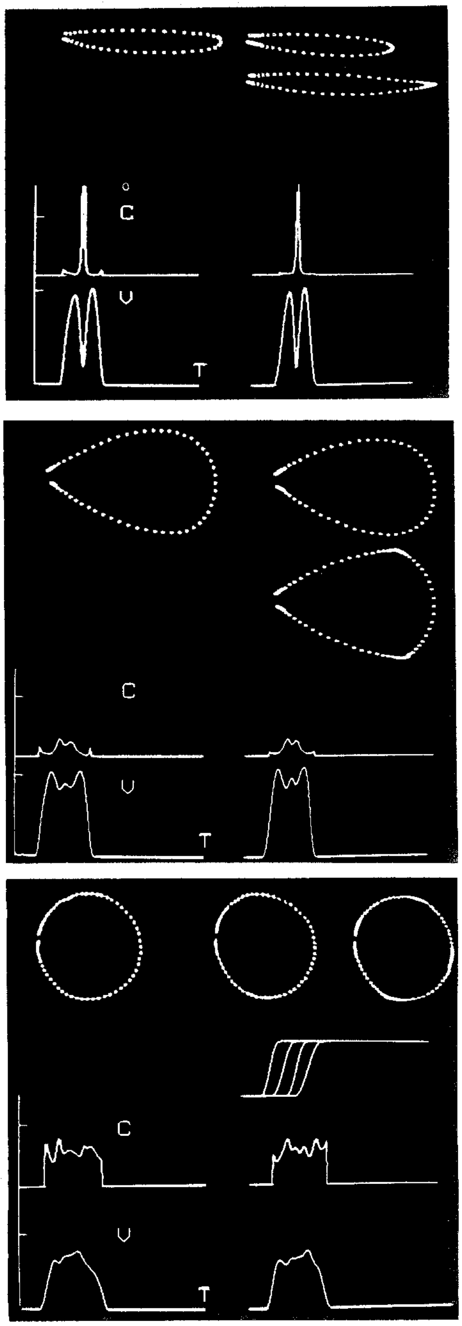

Fig. 6. Closed trajectories. For each frame, the left part shows the experimental trajectory, together with the velocity profile (*V*) and the curvature profile (*C*); the right part shows the model-generated trajectory and the underlying chain of strokes. Velocity calibration: 500 mm/s. Curvature calibration: 4×10^{-2} mm^{-1}. Time calibration: 3.5 s (top), 4 s (center and bottom)

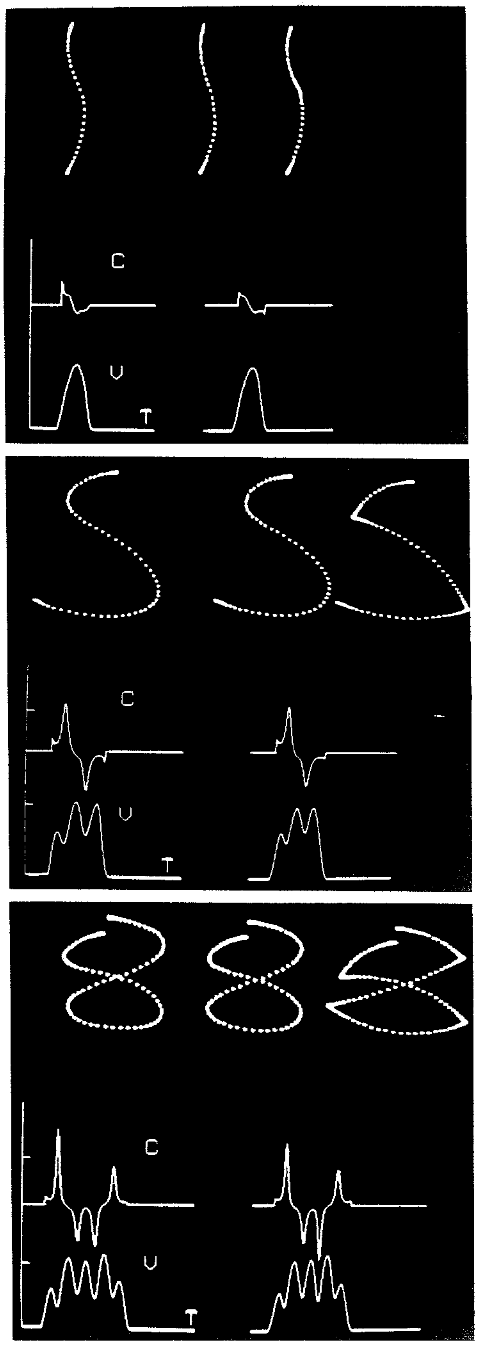

Fig. 7. Trajectories with inflection points. For each frame, the left part shows the experimental trajectory, together with the velocity profile (*V*) and the curvature profile (*C*); the right part shows the model-generated trajectory and the underlying chain of strokes. Velocity calibration: 500 mm/s. Curvature calibration: 8×10^{3} mm^{-1}. Time calibration: 2.6 s (top), 3.2 s (center), 3.6 s (bottom)

velocity peaks. Let us consider two limit cases: narrow closed trajectories have usually only two well distinct velocity peaks, with an associated curvature peak (Fig. 6, top), whereas rounded trajectories have at least four or five less distinguishable velocity peaks (Fig. 6, bottom).

Figure 6 also shows the result of fitting the model to the experimental data. These results agree, in general terms, with the expected effect of quasi-parallel strokes with regard both to the greater irregularity of the recorded pattern and to the greater difficulty to accurately fit the model. At the same time, they suggest

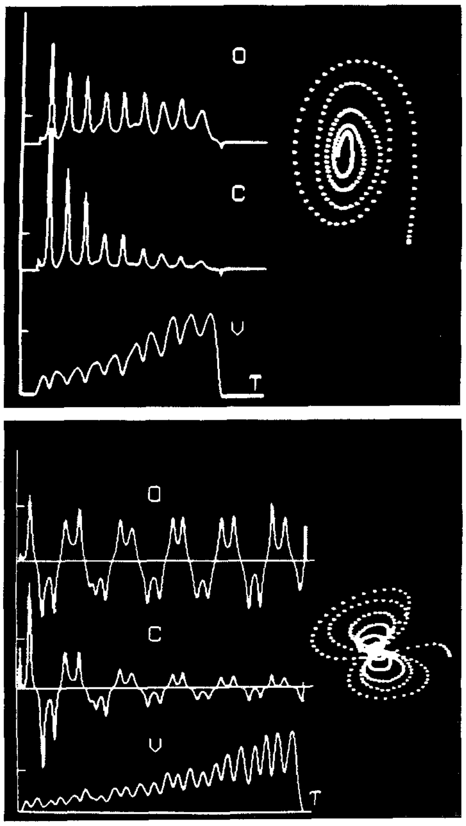

Fig. 8. Spiral trajectories. V: velocity (calibration 500 mm/s); C: curvature (calibration $8 \times 10^{-3}\,\mathrm{mm}^{-1}$); 0: angular velocity (calibration 8 rad/s)

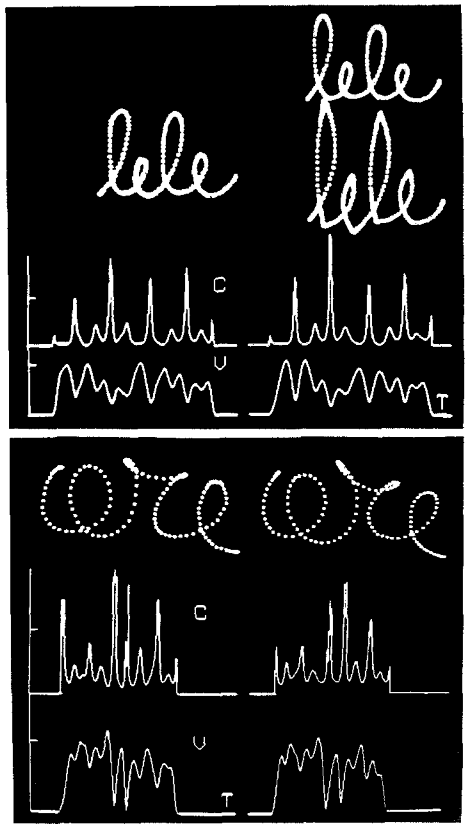

Fig. 9. Handwritten trajectories. For each frame, the left part shows the experimental trajectory, together with the velocity profile (V) and the curvature profile (C); the right part shows the model-generated trajectory and the underlying chain of strokes. Velocity calibration: 500 mm/s. Curvature calibration: $8 \times 10^{-3}\,\mathrm{mm}^{-1}$. Time calibration: 4.5 s

that the motor controller tends to break a trajectory into a number of strokes when the required total angular change (i.e. the angle between the initial and final direction) exceeds 90 degrees.

3.3. Trajectories with Inflection Points

An inflection point is a point where the curvature sign changes and it determines, as a consequence, an *s*-shape.

Figure 7 shows some experimental data related to this situation, together with the fitted model. In the simplest cases, an *s*-shape can be generated (using the stroke composition mechanism) by two different chaining techniques:

i) a counterclockwise (ccw) stroke, followed by a clockwise (cw) stroke (if the "*s*" is generated in the up/down direction) with a small angle at the jointing point;

ii) a sequence of three strokes (ccw + cw + cw or ccw + ccw + cw) with greater angles at the jointing points.

According to the findings of the previous section, we may expect that the former strategy is adopted for rather narrow *s*-shapes (which require a small angular change for each stroke), resulting in a rather symmetric trajectory. The latter strategy, on the other hand, is likely to be used for broader *s*-shapes (which are inevitably slightly asymmetric) and it may be the basic pattern for building a more complex trajectory, such as an 8-shape.

3.4. Spiral Trajectories: Expanding the Size Factor

It is known that the geometric and kinematic characteristics of handwriting are rather invariant with respect to variations of the size scale. It is interesting to explore this property in the general context of trajectory formation, in relation to basic composite patterns, like 1-shapes, 0-shapes, 8-shapes, etc.

For this purpose, we asked the subjects to generate spiral patterns with an 1-kernel, 0-kernel, 8-kernel, etc., such as those shown in Fig. 8. For the recorded data, in addition to the velocity V and the curvature C, we

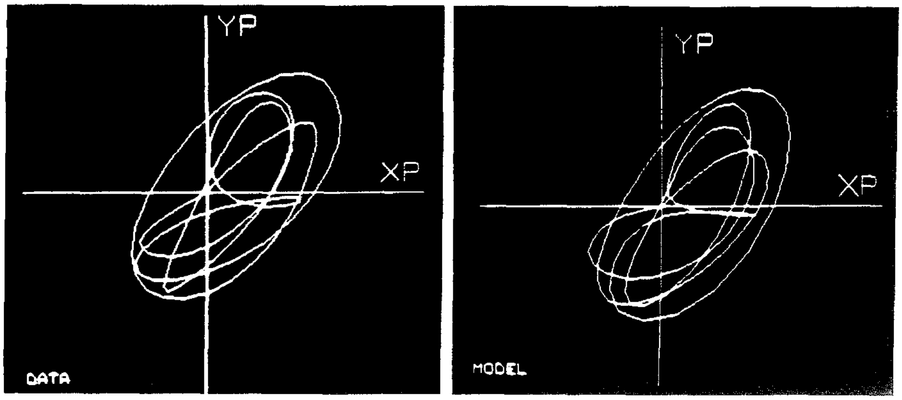

Fig. 10. Velocity space diagram for the handwritten word "are", whose trajectory and velocity/curvature profiles are shown in Fig. 9, bottom. XP, YP: time derivatives of the x and y components of the trajectories. Left: data, right: model

examined also the angular velocity 0 (which can be computed as $0 = VC$).

The recorded trajectories of Fig. 8 (the movements were generated from inside to outside) show that the velocity trace is a sequence of peaks of increasing height, the curvature is a sequence of peaks of decreasing height, and the angular velocity is a sequence of peaks and valleys around a rather constant value. The recorded curves agree with the data of Teulings and Thomassen, but rather loosely with the data of Viviani and Terzuolo (the angular velocity does not seem to be "piecewise constant", unless we consider the average angular velocity).

3.5. *Handwritten Trajectories*

In handwriting, it is possible to find straight strokes (e.g. the upper case "*i*", the crossing stroke of the "*t*"), closed pattens of different roundness ("*e*", "0", etc.), similar patterns of different size ("*e*", "*l*"), patterns with inflection points ("*s*", "*f*", etc.). In a way, it is not surprising that handwritten trajectories have the same geometric/kinematic structure of other simpler movements, as it is shown in Fig. 9, together with the performance of the fitted model.

We also tested our model examining a quite different representation of the handwriting process, suggested by Hollerbach in the context of an oscillatory theory of handwriting. This representation consists of a velocity space diagram, in which the velocity profiles of the x and y components are plotted one against the other. Fig. 10 shows that an experimental handwritten word ("are") and the model generated trajectory exhibit similar diagrams.

4. Discussion

Trajectory formation is a polyedric motor problem which can be studied from many points of view. In particular, we focus on those aspects which involve modeling techniques and artificial intelligence.

With regard to modeling, an important consideration is that trajectory formation models must guarantee sufficient smoothness conditions, which are not met by most existing models of handwriting. For example, the model by Denier Van der Gon and Thuring (which uses a step-wise constant profile of muscle forces) generates piecewise parabolic trajectories which are bound to show curvature discontinuities; the model by Hollerbach (which hypothesizes a piecewise constant modulation of the phase angle of a basic cycloidal pattern) can generate both velocity and curvature discontinuities. On the other hand, the "isogony principle" proposed by Viviani and Terzuolo (even if it were really detectable from the data, which instead show possibly a weaker "average isogony") can hardly be considered a model of trajectory formation: it is descriptive rather than generative, as it assumes that the trajectory is already globally available in some way, somewhere.

In general, any model which addresses explicitly the problem of explaining the segmented nature of handwriting and trajectory formation must be piecewise constant or discontinuous at some level. Our stroke composition model is generative and it is discontinuous at the level of description and planning (the polygonal curve which underlies the actual trajectory is obviously discontinuous and has a discrete character). The global smoothness and continuity of the generated trajectory is an implicit consequence of the composition mechanism (i.e. it is not explicitly planned).

Two other implicit consequences of the structure of the model are:

i) the coupling between the velocity and the curvature profiles,

ii) the "average isogony" that we observed for spiral patterns.

The latter conclusion can be derived in a straightforward way if we only admit that the motor controller uses a "size invariance" principle (i.e. the same timing and kinematic pattern is used for the same trajectory generated according to different size factors).

Turning now our attention to the artificial intelligence point of view, we would like to remark, as Sutherland (1979), that many of the tasks controlled by the human brain are so complex that the only way to achieve a theoretical representation is to build computational models. Very good examples of this approach are the theories developed by Marr and his colleagues on early processing in human vision (Marr, 1976, 1980) and on stereo vision (Marr and Poggio, 1979). In the field of motor control the state-of-the-art is perhaps less advanced, one reason being that most available data involve single degree of freedom movements, with the exception of a limited number of studies (e.g. Polit and Bizzi, 1979; von Hofsten, 1980; Morasso, 1981; Abend et al., 1982). On the other hand, most theoretical studies on motor control issues (e.g. Bernstein, 1957; Gelfand et al., 1971; Greene, 1972; Arbib, 1975; Goodnow and Levine, 1973) are more descriptive and speculative than capable of providing computational paradigms.

The trajectory formation model which has been proposed here is an attempt to define a generative structure that, on one side, agrees with the experimental data and, on the other side, provides a concise description of movements which is compatible with other levels of neural processing, such as the visual level and the cognitive level. Using the Artificial Intelligence language, we can naturally define LISP-like descriptors of strokes of the following type:

[stroke name (CLASS STROKE)
(TILT ANGLE *a*)
(LENGTH $\underline{l}$)
(ANGULAR CHANGE *f*)
(DURATION *t*)
(DELAY *d*)]

and we like to remark the formal and conceptual similarity of this type of descriptor with the descriptors proposed by Marr and Hildreth (1979) to denote the "edge" (i.e. a basic element of scene analysis):

[edge name (CLASS EDGE)
(TILT ANGLE *a*)
(LENGTH $\underline{l}$)
(POSITION *xy*)]

A description of movements in terms of strokes could also be used as a formal method of movement notation, in addition to or in alternative with other methods (e.g. Eshkol, 1958; Huthchinson, 1970; Golani, 1976; Lansdown, 1978) which address the problem of representing global body movements in a rather simplified way. The advantage of a stroke notation is that it needs only to store geometric parameters (except for a specification of the time scale, i.e. the "tempo" of the trajectories) because timing is implicitly provided by the standard stroke generating functions.

Another way to look at the stroke representation of movement is from the point of view of information theory. This description results in a significant compression of the information needed to store a complex movement, in a similar way to the description/codification of speech which uses the method of formants[2]. Within the limits of an analogy, the role of the tunable resonant cavities is played by the stroke generators and the role of the formant parameters is played by the stroke parameters.

In a way, it is surprising that so different motor acts (from the cognitive point of view) as point-to-point movements and handwriting have similar kinematic characteristics and can be fitted by means of the same trajectory formation model. However, it can be argued that the role of the model is just to provide a uniform vocabulary to a "motor semantic" level of a motor control. This could motivate the search of cognitive structures in the sequences of strokes generated in response to given behavioral situations, starting, for example, with simple statistics and clustering of stroke parameters.

Appendix A: *B*-Splines and *S*-Splines

A standard way to represent a planar curve with continuous velocity and curvature profiles consists of using cubic spline functions:

$$\underline{r}=\underline{r}(t)=\underline{r}_1 B_1(t)+\ldots+\underline{r}_n B_n(t), \tag{A.1}$$

where $\underline{r}_1, \ldots, \underline{r}_n$ are constant vectors and $B_1(t), \ldots, B_n(t)$ are cubic *B*-spline functions (de Boor, 1979). The *B*-splines have limited support (i.e. they are "bell-shaped") and, in the cubic case, only four *B*-splines are active at any time instant. Therefore, (A.1) can be interpreted geometrically if we consider $\underline{r}_1, \ldots, \underline{r}_n$ as the verteces of a polygonal curve and we say that, for each time instant, the actual curve is shaped under the action of four consecutive verteces.

Alternatively, we may focus our attention on the sides of the polygonal curve $((\underline{r}_{i+1}-\underline{r}_i),\ i=1,\ n-1))$ instead of the verteces $(\underline{r}_i,\ i=1, n)$ and we end up with the following interpolation formula

$$\underline{r}=\underline{r}(t)=\underline{r}_1+(\underline{r}_2-\underline{r}_1)S_1(t)+\ldots(\underline{r}_n-\underline{r}_{n-1})S_{n-1}(t), \tag{A.2}$$

2 If we assume a sampling rate of 50 samples/s and a stroke rate of 2 stroke/s, the compression factor is $(2\times 3)/(2\times 50)=6/100$

where it is possible to verify that the functions $S_i(t)$ (we may call them S-splines) are given by

$$S_i(t)=\sum_{i+1}^{n}{}_j\, B_j(t) \qquad (i=1,n-1) \tag{A.3}$$

simply substituting (A.3) in (A.2) and getting back (A.1) $\left[\text{remember that } \sum_{1i}^{n} B_i(t)=1\right]$.

It should be noticed that cubic B-splines are bell-shaped and the cubic S-spline are step-shaped. Furthermore, the cubic B-splines have a time support of four time intervals, whereas the cubic S-splines have a transient of only three time intervals (Fig. 1A shows a set of S-splines with the corresponding B-splines). In other words, for each time instant the actual curve is shaped by three consecutive sides of the polygonal curve and it comes close to the middle of a side when the corresponding S-spline reaches its mid-course.

Appendix B: Sensitivity Analysis of the Model and a Fitting Algorithm

In order to fit he generative model of Sect. 2 to experimental data, it is necessary to take into account the trajectory, the velocity profile, and curvature profile. On the other hand, the parameters which need to be determined, for each stroke, are the length, the tilt, the curvature, and the timing (delay and duration).

If we consider that even apparently simple trajectories may contain tens of strokes, it is clear that fitting in general is a quite complex problem. However, it is clear that the different parameters play a different role in the generation process and this suggests to perform a sensitivity analysis, by means of computer simulation, for sorting the main effects. Here we list the main conclusion of this analysis:

Influence of Stroke Parallelism

If two consecutive strokes are nearly parallel (i.e. if the angle between the final tangent of the previous stroke and the initial tangent of the following stroke is small), the two velocity peaks tend to blur into a single smooth dome-shape. A prediction which can be made from this fact is that those trajectories which derive from a sequence of nearly parallel strokes (sequences of circles, spirals etc.) are likely to show a much more "irregular" velocity profile than those trajectories which derive from a sequence of highly non parallel strokes (e.g. a sequence of handwritten l's).

Influence of Stroke Timing

If we change the timing parameters (time-shift and duration) of a stroke, it is possible to see that even very small changes influence the velocity profile significantly, whereas the generated shape is virtually unaffected. We also found that this effect is particularly strong for nearly parallel strokes.

Influence of Stroke Curvature

On the contrary, if we change the curvature of a stroke, we alter directly the trajectory whereas the velocity profile is little affected.

Summing up, we can say that the geometric parameters of the stroke influence more strongly the trajectory and the curvature profile, whereas the timing parameters influence more strongly the velocity profile.

A fitting procedure can take advantage of these effects and, accordingly, we devised an interactive algorithm which consists of the following steps:

1. determine the trajectory points which correspond to a peak of the velocity profile and to a dip or an inflection of the curvature and store the corresponding time instants,
2. draw the circles tangent at each determined point with a radius of curvature given by the value of the curvature profile at the selected time instants,
3. determine the intersections between consecutive circles (these are taken as terminal points of each stroke),
4. assume a tentative time overlap pattern like that of Fig. 1-bottom (i.e. if t_1 is the initial time instant and $t_2, t_3, \ldots$ are the time instant which correspond to the first, the second, velocity peaks, assume that the first stroke begins at t_1 and ends at t_3, the second stroke begins at t_2 and ends at t_4, and so on),
5. compare the experimental and the simulated velocity and curvature profiles and single out which peaks have wrong height or melt incorrectly with neighbouring peaks,
6. repeatedly adjust the timing patterns of the corresponding strokes until an acceptable sequence of peaks and dips is obtained.

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References

Abend, W., Bizzi, E., Morasso, P.: Human arm trajectory formation. Brain **105**, 331–348 (1982)

Arbib, M.A.: Artificial Intelligence and brain theory: unities and diversities. Ann. Biomed. Eng. **3**, 228–274 (1978)

Benati, M., Gaglio, S., Morasso, P., Tagliasco, V., Zaccaria, R.: Anthropomorphic robotics. I. Representing mechanical complexity. Biol. Cybern. **38**, 125–140 (1980)

Benati, M., Morasso, P., Tagliasco, V.: The inverse kinematic problem for anthropomorphic manipulator arms. Trans. ASME, J. Dyn. Syst. Measur. Cont. (in press)

Bernstein, N.: The coordination and regulation of movements. London: Pergamon Press 1967
Boor, C. de: A practical guide to splines. Berlin, Heidelberg, New York: Springer 1978
Crane, H.D., Savoie, R.E.: An on-line data entry system for handprinted characters. Computer, March, 43–50 (1977)
Denier van der Gon, J.J., Thuring, J.Ph.: The guiding of human writing movements. Kybernetik **2**, 145–148 (1965)
Eden, M.: Handwriting generation and recognition. In: Kolers, P.A., Eden, M. (eds.). Recognizing patterns, pp. 138–154. Cambridge, Mass: MIT Press 1968
Eshkol, N., Wachman, A.: Movement notation. Weidenfeld and Nicolson (1958)
Faux, I.P., Pratt, M.J.: Computational geometry for design and manufacture. Chichester (West Sussex): Ellis Horwood Ltd 1979
Galen, G.P. van: Handwriting and drawing: a two stage model of motor behaviour. In: Stelmach, G.E., Requin, J. (eds.). Tutorials in motor behaviour, pp. 567–578. Amsterdam: North-Holland 1980
Gelfand, I.M., Gurfinkel, V.S., Tsetlin, M.L.: Models of the structural-functional organisation of certain biological systems. Cambridge, Mass: MIT Press 1971
Golani, I.: Homeostatic motor processes in mammalian interactions: a choreography of display. In: Bateson, P.P., Kofler, P.H. (eds.). Perspective in ethology. Vol. 2. New York: Plenum Press 1976
Goodnow, J.J., Levine, R.A.: The grammar of action: sequence and syntax in children's copying. Cogn. Psychol. **4**, 82–98 (1973)
Greene, P.H.: Problems of organisation in motor systems. In: Rosen, R., Snell, F.M. (eds.). Progress in theoretical biology, Vol. 2. New York: Academic Press 1972
Herbst, N.M., Liu, C.N.: Automatic signature verification based on accelerometric data. I.B.M. J. Res. Develop., May, 245–253 (1977)
Hofsten, C. von: Predictive reaching for moving objects by human infants. J. Exp. Child Psychol. **30**, 369–382 (1980)
Hogan, N.: Mechanical Impedance Control in assistive devices and manipulators. Proc. Joint Autom. Control Conf. **1**, TA10-B (1980)
Hollerbach, J.M.: An oscillation theory of handwriting. Biol. Cybern. **39**, 139–156 (1981)
Hutchinson, A.: Labanotation. New York: Theater Arts Books 1970
Katz, D.: Gestalt Psychology. London: Methuen 1951
Lashley, K.S.: The problem of serial order in behaviour. In: Jeffress, L.A. (ed.): Cerebral mechanisms in behaviour, pp. 112–146. New York: Wiley 1951
Lansdown, J.: The computer in choreography. Computer, August, 19–30 (1978)
Marr, D.: Early processing of visual information. Trans. R. Soc. (London) **275**, 483–524 (1976)
Marr, D., Hildreth, E.: Theory of edge detection. A.I. Memo N. 518 (1979)
Marr, D., Poggio, T.: A computational theory of human stereo vision. Proc. R. Soc. (London) **204**, 301–328 (1979)
Marr, D.: Visual information processing: the structure and creation of visual representation. Philos. Trans. R. Soc. (London) **290**, 199–218 (1980)
Mermelstein, P., Eden, M.: Experiments on computer recognition of connected handwritten words. Inf. Cont. **7**, 225–270 (1964)
Morasso, P.: Spatial control of arm movements. Exp. Brain Res. **42**, 223–227 (1981)
Morasso, P., Mussa Ivaldi, F.A.: Un modele de generation des mouvements d'écriture. Journees Thematiques, Forum Espace III, Position et Mouvements, Marseille (1981)
Polit, A., Bizzi, E.: Characteristics of the motor programs underlying arm movements in monkeys. J. Neurophysiol. **42**, 183–144 (1979)
Sutherland, N.S.: The representation of three-dimensional objects. Nature **278**, 395–398 (1974)
Teulings, H.L.H.M., Thomassen, A.J.W.M.: Computer aided analysis of handwriting movements. Visible Language; XIII **3**, 218–231 (1979)
Viviani, P., Terzuolo, C.A.: Space-time invariance in learned motor skills. In: Stelmach, G.E., Requin, J. (eds.). Tutorials in motor behaviour, pp. 525–533. Amsterdam: North-Holland 1980
Vredenbregt, J.J., Koster, W.G.: Analysis and synthesis of handwriting. Philips Tech. Rev., **32**, 73–78 (1971)

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