Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
Human Movement Science 24 (2005) 731–743
HUMAN MOVEMENT SCIENCE
www.elsevier.com/locate/humov

Variations in the relationship between radius of curvature and velocity as a function of joint motion

Lauren L. Saling *, James G. Phillips

Psychology Department, Monash University, Victoria 3800, Australia

Available online 11 October 2005

Abstract

A power law describes the relationship between the geometric properties of a trajectory (radius of curvature) and movement kinematics (tangential velocity) in curved drawing movements. Although the power law is a general law of motion, there are conditions under which it degrades. In particular, the power law may be less explanatory of movements around certain joints. The present study considered how varying motion around different joints influenced the fit of the power law. Motions associated with finger and wrist, or elicited by an isometric force production task, were compared. The power law was most explanatory of finger motion and isometric production and least explanatory of wrist motion. The fit of the power law for finger and wrist motion suggested separate laws for each joint system. Since the fit of the power law was better for finger than for wrist motion, there is some suggestion that the power law better explains motion around fewer or simpler joint systems.

PsycINFO classification: 2330

Keywords: 1/3 power law; Joint motion; Trajectory formation; Coordination

1. Introduction

Hand trajectories are produced with regularity despite the complexities inherent in a system with so many degrees of freedom. This suggests that there are organising principles

* Corresponding author. Tel.: +61 3 9905 3939; fax: +61 3 9905 3948.
E-mail address: Lauren.Saling@med.monash.edu.au (L. Saling).

0167-9457/$ - see front matter
doi:10.1016/j.humov.2005.09.001

inherent within the motor system that constrain redundant degrees of freedom. There are invariant aspects to the manner in which motor tasks are completed, and these effectively reduce the number of degrees of freedom associated with the system involved in movement production. Invariances potentially provide an insight into the fundamental organising principles of motor control and inform our understanding of coordinative processes (Schmidt, 1984).

1.1. The degrees of freedom problem

Bernstein (1967) noted that there is extensive redundancy within the motor system, owing to excessive degrees of freedom. This conveys extensive movement flexibility in that there an infinite number of ways in which a motor task can be completed, but poses a problem for the control of movement. The degrees of freedom problem refers to the paradox associated with the fact that, in spite of the extensive redundancy in the motor system, motor performance occurs with regularity (Bernstein, 1967). Coordination involves constraint of degrees of freedom to reduce the redundancy inherent within the motor system. Solutions to the problem of excess degrees of freedom tended to take the form of optimisation functions which maximised movement efficiency, and this was used as an argument to dismiss computational accounts (see, e.g., Kelso, 1995). Potential invariances serve to illustrate how such problems are solved. One such invariance is the power law.

1.2. The power law of circular drawing motion

A reliable relationship has been noted between tangential velocity and the radius of curvature in curved drawing movements (Lacquaniti, Terzuolo, & Viviani, 1983). The power law dictates that sharper curves are drawn at slower speeds. This relationship holds for a wide variety of trajectories, from ellipses to complex geometrical shapes as well as being robust to manipulations of size and movement direction (Lacquaniti et al., 1983, Lacquaniti, Terzuolo, & Viviani, 1984; Viviani & Flash, 1995). Hence the power law constitutes an invariance principle. This study addresses factors contributing to the power law. In particular, the study will investigate the role of limb motion, and any influence of variations in the joints used, upon the fit of the power law.

The relationship between velocity and curvature has been variously described as the 1/3 or 2/3 power law, however, the two are mathematically equivalent. In the 2/3 power law, angular velocity is substituted for tangential velocity and the exponent becomes 2/3. The general formula for the power law is $V(t) = kR(t)^{\beta}$, where V is tangential velocity, k is a proportionality constant, R is the radius of curvature, and β is a constant which becomes 1/3 in adulthood (Sciaky, Lacquaniti, & Terzuolo, 1987; Viviani & Flash, 1995; Viviani & Schneider, 1991). The power law has been described as a simple law of motion which is generally robust to experimental manipulation (Lacquaniti et al., 1983, 1984; Viviani & Flash, 1995). There are conditions, however, under which the power law degrades or is less explanatory of trajectory formation.

Since its inception, various explanations of the power law have been offered. In particular, two competing accounts of the power law, the dynamical/ecological account and the computational account, have emerged. Invariance implies constraint; however this does not speak to whether it is computed or self-organised.

Table 1
Summary of empirical observations supporting either the computational or dynamic account

Manipulation	Finding	Account
Complex trajectories	Power law held	Dynamic
Timing cues	Power law held	Dynamic
Aging study	Degrades with age	Dynamic
Speed	Degrades at slower speeds	Dynamic
Isometric device	Power law held	Computational
Robot arm	Power law held	Computational
Developmental study	Strengthens with age	Computational
Joints	Better explains some joints	Computational

In order to succeed as mechanisms underpinning the power law, the computational and dynamical/ecological accounts need to explain the existing body of knowledge concerning the power law including conditions under which the power law degrades. The computational and dynamical systems accounts of the power law are supported by different empirical observations. Table 1 lists empirical observations associated with the power law.

1.3. The dynamical/ecological account of the power law

Dynamical/ecological models invoke emergent self-organising properties of moving muscles and joints. Theorists such as Kelso (1995) assert that tasks including trajectory formation are performed by a pattern forming, self-organised system. As such, there is no reliance upon computation. This is analogous to the situation with slide rules, where the solution is an inherent part of the structure of the device. The postulated peripheral mechanisms, which are thought to be responsible for invariances such as the power law, include a movement system which promotes smoothness of the hand trajectory (Todorov & Jordan, 1998), limb dynamics (Gribble & Ostry, 1996) and sinusoidal joint motion (e.g., Schaal & Sternad, 2001).

Dynamical/ecological accounts are supported by the observation that the power law exists irrespective of the complexity of the curved trajectory (Lacquaniti et al., 1983, 1984; Viviani & Flash, 1995). This relationship has been found to hold for a wide variety of trajectories, from ellipses to complex geometrical shapes including clover leaves. In addition, our research demonstrates that the power law is unaffected by the availability of timing cues suggesting that the operation of the power law is not dependent upon a cognitive mechanism (Saling & Phillips, 2002). Further support for a dynamical account comes from the observation that the power law is less explanatory of trajectories performed at slower speeds (Wann, Nimmo-Smith, & Wing, 1988). Additionally, our research indicates that the power law degrades with advancing age, possibly suggesting a deterioration of peripheral processes (Saling & Phillips, 2002), which supports a dynamical/ecological model of the power law.

1.4. The computational account of the power law

In contrast to the self-organising nature of motor behaviour on the dynamical account, proponents of computational models invoke control by a cognitive or neural mechanism. This suggests that the power law arises from an organisational principle in the motor system related to central constraints for the production of a motor trajectory and is not a muscular property. The computational account of the power law requires that movement kinematics are centrally represented and that trajectory formation is centrally planned prior to execution (e.g., Flash & Hogan, 1985). Hand trajectories are planned in hand coordinates or joint coordinates such that a particular strategy for hand motion can be selected from the infinite available strategies (Flash & Sejnowski, 2001).

Support for a computational account of the power law is found developmentally, neurally, and in some puzzling biomechanical observations. Developmental studies of the operation of the 1/3 power law have revealed that this relationship emerges and strengthens with maturation of the nervous system, although it is present in children as young as 3 years, albeit in a weakened form (Sciaky et al., 1987; Viviani & Schneider, 1991). Additionally, there has been some data collected in the context of monkey trajectory formation involving single cell recording which suggests a neural basis for the power law (see Georgopolous, Kettner, & Schwartz, 1988).

In addition, the necessary contribution of moving muscles has been questioned. The operation of the power law has been investigated where trajectories are completed using two- and three-dimensional isometric devices and a robot arm where the role of muscle properties is less cogent (Massey, Lurito, Pellizer, & Georgopoulos, 1992; Pellizer, Massey, Lurito, & Georgopolous, 1992; Schaal & Sternad, 2001). In the case of the robot arm, no muscles are involved in movement completion. Where an isometric device is used, there is essentially no limb movement involved but rather force production. Although it must be noted that just because the device is not moving, it does not necessarily follow that the muscles are not moving (Ito, Kawakami, Ichinose, Fukahiro, & Fukunaga, 1998). Studies incorporating a robot arm and isometric devices have revealed that the power law is explanatory of trajectories completed with no limb movement and where muscular movement is less cogent. The findings of such studies lend support for a computational account of the power law. It is also of interest that the power law has been shown to hold for foot trajectories, in particular the swing phase of human locomotion (see Ivanenko, Grasso, Macellari, & Lacquaniti, 2002) and a relationship resembling a 1/3 power function has been noted in speech production (see Tasko & Westbury, 2004). Owing to the quite disparate muscle and joint properties of the hand, foot and articulatory systems, the observation of this lawful relationship in foot motion and speech production strains the credulity of a peripheral account of the phenomenon.

Recently the explanatory nature of the power law has been noted to vary over particular joints, and this may support a computational account of the power law as it suggests that different joints carry a different computational burden. Our research has revealed that movement around the shoulder and elbow joints appears to be better accounted for by the power law than does movement around the wrist joint (Saling & Phillips, 2002). And as proximal joints potentially have a greater impact upon movement endpoint outcome than distal joints, then it implies the power law is a function of proximal joints.

In light of the fact that there is support for both accounts of the power law, further work is needed to explicate which approach better underpins the power law.

1.5. Goodness of fit of the power law varies over different joints

Saling and Phillips (2002) investigated the operation of the power law with advancing age as a function of joint motion (peripheral manipulation), speed of trajectory formation,

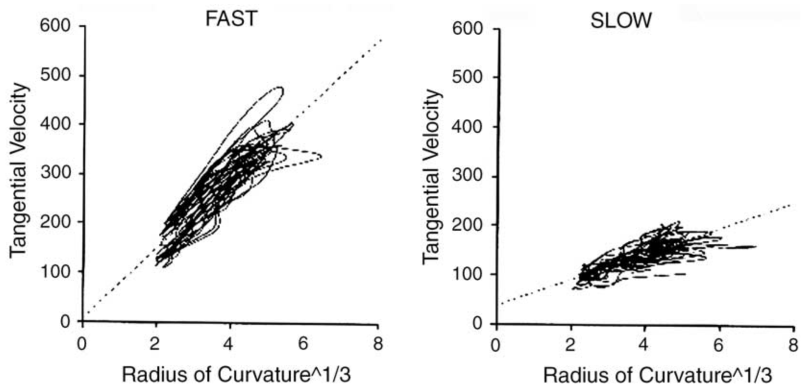

Fig. 1. Representative single trials for the fast (2 Hz) and slow (1 Hz) conditions.

and a timing cue (cognitive manipulation). Participants completed ellipses using both a wrist motion and a shoulder motion. Ellipses were completed at both a fast speed (2 Hz) and a slow speed (1 Hz). A timing cue (metronome) was either present or absent. The proportion of variance accounted for by the power law in a particular experimental condition determined the fit of the power law.

The manipulation of speed affected the operation of the power law. In particular, the power law degraded at slower speeds. Fig. 1 depicts representative single trials for a young adult in the fast condition and a young adult in the slow condition. The power law was found to be robust to the provision of timing cues as it was unaffected by the presence or absence of a metronome. The power law degraded with advancing age as a function of joints used in trajectory formation.

The power law was found to be more explanatory of trajectories of certain joints than others. When using a shoulder motion the power law accounted for 62% of the variance, while only 27% of the variance was accounted for by the power law when using a wrist motion. Fig. 2 depicts representative single trials for a young adult in the shoulder condition and a young adult in the wrist condition.

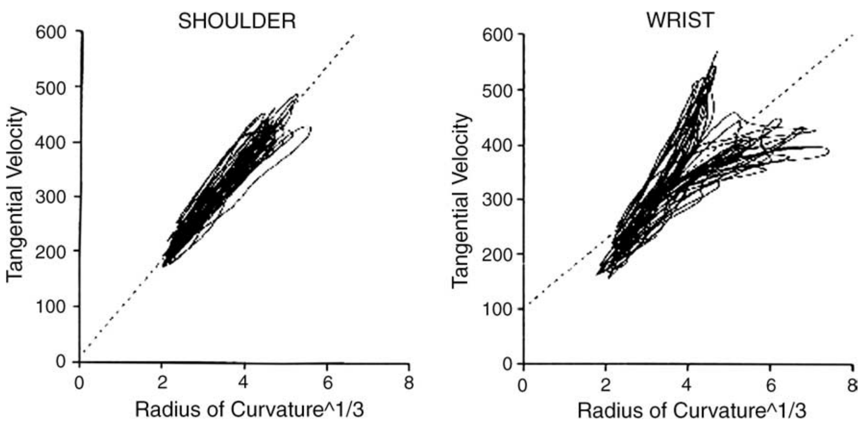

Fig. 2. Representative single trials for the fast proximal (shoulder) condition and the fast distal (wrist) condition.

2. The present study

The results of our previous study suggest that the power law better describes motion around proximal joints than distal joints as the power law was more explanatory of movements performed using the shoulder motion than those performed using the wrist motion. Therefore, there appears to be a need to further examine the impact of joints used in movement completion on the fit of the power law. The present study seeks to replicate and extend our previous finding, considering whether the power law applies for other more distal joints (finger joint) and whether the power law applies where there is no motion.

2.1. Participants

Seven right-handed volunteers participated in the study (mean age = 30.57 years, SD = 7.30 years). There were three males and four females. Experiments were conducted according to ethical guidelines specified by the Monash University Ethics Committee.

2.2. Apparatus and task

The task involved participants moving a cursor on a computer screen repetitively around an elliptic template (long axis = 10.2 cm, short axis = 5.1 cm, eccentricity = 0.866), in a clockwise direction. The elliptic template was oriented at a 45° angle to facilitate drawing. Ellipses on the computer screen were completed under three experimental conditions. Cursor motion was controlled by: a wrist motion; a finger motion; or by isometric forces generated at the finger tip. Trajectory formation was self-paced.

2.3. Design

There was a repeated measures design with all participants completing ellipses under the three experimental conditions (wrist, finger and isometric conditions). The order of presentation of the three experimental conditions was randomised.

2.4. Procedure

The task was demonstrated by the experimenter and participants performed 15 practice trials each of 20 s duration. Participants completed all ellipses with their right hand using either a wrist motion, a finger motion or isometric force production with the order of presentation randomised. The device used for the wrist condition was a mouse. The wrist motion was achieved by participants resting their wrist on the desk and restricting any shoulder motion. As such, wrist and hand motion moved the cursor on the screen. The device used for the finger condition was a touchpad. The finger motion was achieved by participants manipulating the touchpad and hence moving the cursor on the screen with only their index finger and restricting any wrist motion. In particular, only the proximal interphalangeal joint was used to move the cursor. An isometric device (an accupoint mouse) operated by the right index finger and not requiring limb motion but only pressure, was used to achieve the isometric force production. As such, changes in pressure caused cursor movement on the screen in the isometric condition. Ten trials of 20 s each were then recorded and analysed for each experimental condition.

2.5. Analysis

Data were analysed using automatic computer algorithms. The x- and y-coordinates on the screen were low-pass filtered (10 Hz cutoff) using a second-order Butterworth filter (with dual pass to remove phase lag), and differentiated using a 9-point central finite differences algorithm using a moving window to obtain tangential velocity and curvature function. The number of submovements was determined by counting the peaks in the tangential velocity function.

The equation for tangential velocity is for point (x_5, y_5):

$$\mathrm{TanVel} = \frac{\sqrt{(x_9 - x_1)^2 + (y_9 - y_1)^2}}{t}.$$

The equation for the radius of curvature is

$$R = \left|\frac{\ddot{x}\dot{y} - \ddot{y}\dot{x}}{(\dot{x}^2 + \dot{y}^2)^{3/2}}\right|.$$

Tangential velocity, curvature and the number of submovements were calculated for every trial (10 trials of 20 s duration for each of the three conditions) for each participant. One way repeated measures ANOVAs were performed to determine whether there were significant differences between finger, wrist and isometric conditions in terms of tangential velocity, fit of the power law, and the number of submovements. When a significant result was obtained using ANOVA, planned comparison (simple contrasts as per SPSS and effectively equivalent to t-tests) were undertaken in order to determine which groups differed significantly from one another.

2.6. Measures

2.6.1. Power law

For adults, the power law predicts a linear relationship between tangential velocity and the radius of curvature, raised to the 1/3 power. To appraise the adequacy of the power law, a least squares regression analysis was used. The proportion of variance accounted for by the 1/3 power law was calculated from the square of the correlation coefficient (r^2), to determine the goodness of fit of the power law.

2.6.2. Submovements

The number of peaks in the tangential velocity function was calculated in order to determine the number of submovements. This measure was used in order to establish whether differences in the number of submovements between different motions used would explain changes in the power law.

2.7. Results

2.7.1. Tangential velocity

As variations in tangential velocity influence the operation of the power law (Saling & Phillips, 2002), speed of trajectory formation was determined. A one way repeated measures ANOVA revealed that there were no significant differences in the speed of trajectory formation between finger motion ($M = 232.18$ mm s^{-1}, SD = 134.28), wrist motion ($M = 240.50$ mm s^{-1}, SD = 152.18) and isometric force production ($M = 238.35$ mm s^{-1}, SD = 159.95), $F(2,12) = .07$, $p > .05$, $\eta^2 = .01$, see Fig. 3. Since there were no significant differences in speed of movement completion, the differences in the goodness of fit of the power law with different motions is not merely an artefact of speed differences.

2.7.2. Goodness of fit

The trials analysed may be described in terms of a 1/3 power law, with the power law explaining on average between 30% and 59% of the variance in individuals' 10 20 s trials. The proportion of variance (r^2) accounted for by the power law was thus analysed.

The operation of the power law was affected by the motion used in trajectory formation, $F(2,12) = 4.75$, $p < .05$, $\eta^2 = .44$. The effect may be seen in Fig. 4 (finger motion, $M = .59$, SD = .15, wrist motion, $M = .30$, SD = .16 and isometric force production $M = .48$, SD = .21). Simple planned comparisons revealed a significant difference between finger and wrist motion, $F(1,6) = 12.91$, $p < .05$, $\eta^2 = .68$. Wrist and isometric force did not differ significantly, $F(1,6) = 2.65$, $p > .05$, $\eta^2 = .31$. Finger motion and isometric force did not differ significantly, $F(1,6) = 1.49$, $p > .05$, $\eta^2 = .20$.

Fig. 5 depicts representative single trials for finger and wrist motions. The goodness of fit of the power law is different for the wrist joint and the finger joint. In particular, the power law better explains motion around the finger joint than motion around the wrist

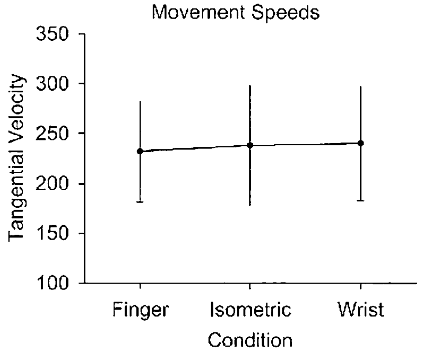

Fig. 3. Tangential velocity (mm s^{-1}) for the three conditions (finger, isometric and wrist).

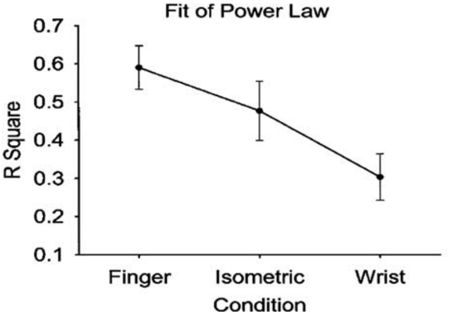

Fig. 4. Proportion of variance accounted for (goodness of fit) by the power law for the finger, isometric and wrist conditions.

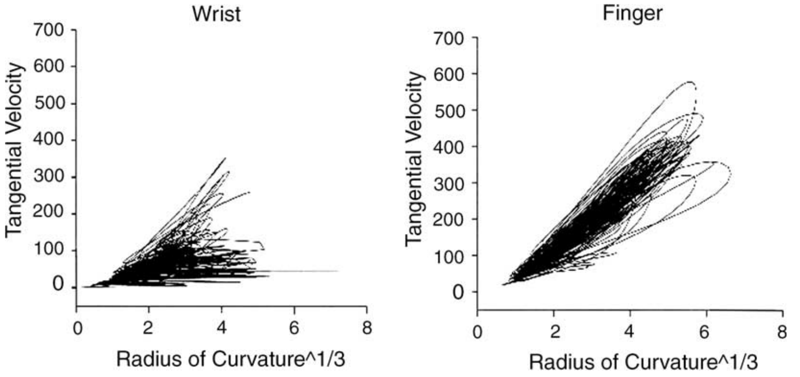

Fig. 5. Representative single trials for wrist and finger conditions.

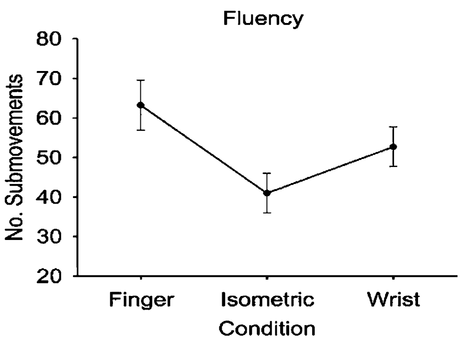

Fig. 6. Number of submovements as a function of condition (finger, isometric and wrist).

joint. Plots of the power law for the wrist are fan shaped, and give the impression that multiple separate power laws are in operation.

2.7.3. Submovements

There were significant differences in the number of submovements between motions employed, $F(2,12) = 7.15$, $p < .05$, $\eta^2 = .54$. This effect may be seen in Fig. 6 (finger motion, $M = 63.25$, SD = 16.78; wrist motion, $M = 52.71$, SD = 13.22; isometric force $M = 40.98$, SD = 13.48). Planned comparisons revealed a significant difference between the finger and isometric conditions, $F(1,6) = 10.52$, $p < .05$, $\eta^2 = .64$. There was no significant difference between wrist and finger conditions, $F(1,6) = 3.75$, $p > .05$, $\eta^2 = .39$ and wrist and isometric conditions, $F(1,6) = 5.03$, $p > .05$, $\eta^2 = .46$. The differences in the number of submovements do not parallel the differences noted in the power law between conditions (see Fig. 4) and as such do not explain the variations noted in the power law as a function of motion employed.

3. Discussion

Invariances, such as the power law, potentially provide insights into motor control. As such, the importance of determining the putative mechanisms underpinning the relationships that such invariances encompass, cannot be overstated. The power law has been described as a simple, general law of motion which is robust to a wide variety of experimental manipulations. In spite of this, there are conditions under which the power law degrades. The present study further investigated the impact of varying joint motion on the operation of the 1/3 power law. In particular, the present study demonstrated that the power law could be a function of distal joints. Furthermore, the present study investigated whether motion is required for the power law.

3.1. The power law and movement

In order to determine whether motion is required for the power law, isometric force production was used for trajectory formation. The power law was explanatory of trajectories produced by isometric force production. Although trajectory formation, which is performed using an isometric device, may still involve muscular contractions (Ito et al., 1998), this finding is inconsistent with dynamical/ecological accounts of the power law, as it reduces the role of peripheral features including muscles and joints. Indeed as limb movement is not required, the power law may reflect central processes, which dictates that movements with smaller radii of curvature are more unpredictable unless taken at slower speeds. This may reflect computation, but at the very least it minimises the role played by muscle motion.

3.2. The power law, speed and submovements

Differences were noted in the fit of the power law for the different motions employed in trajectory formation. There were no significant differences in speed of trajectory completion, suggesting that speed differences do not account for the explanatory ability of the power law using different motions. Furthermore, submovements do not readily account for goodness of fit of the power law as there were no significant differences in the number of submovements when comparing the wrist and finger conditions. Goodness of fit and fluency of movements did not covary, either in the present study, or in Saling and Phillips (2002), and so intermittency of control is not a cogent explanation of the data.

3.3. The power law and joint motion

Our findings suggest that joint complexity does play a role in the explanatory ability of the power law as the fit of the power law was different for different joint motions. The proportion of variance accounted for by the power law was greatest for distal joints, finger motion, suggesting that simple joints explain the fit of the power law. This confirms our earlier findings (Saling & Phillips, 2002) that the power law provides less certain solutions when the joints involved in movement completion are more complex. Of interest is that our earlier study suggested that the power law better described motion around proximal joints (shoulder, $r^2 = .62$) than distal joints (wrist, $r^2 = .27$) whereas in this study, the power law was less explanatory of the more proximal joint (wrist, $r^2 = .30$) than the distal joint (finger, $r^2 = .59$). By constraining and decomposing curved drawing movements into contributing joints it appears that the power law is probably a phenomenon that occurs over a simple joint. In previous studies, proximal joints have probably made the major contribution to output, swamping the contributions of the more distal joints. But the present study shows that the power law is not merely a product of proximal joints, it also applies to distal joints. This suggests that the power law is more a function of simple joints.

When the motor system is presented with a variety of joints, as in the wrist system, the power law degrades. This is probably because of the greater number of degrees of freedom associated with the wrist joint as compared to the finger joint. Teulings, Thomassen, and Maarse (1989) observed for the hand/wrist system that motion occurred at different speeds in different directions, presumably as a function of finger and wrist motion. As the power law varies in strength as a function of movement speed (Saling & Phillips, 2002; Wann et al., 1988), it is possible that the degradation of the power law might reflect the mixture of slower finger and faster wrist motions. However, this is unlikely. There were no significant differences in tangential velocity between finger and wrist conditions. Indeed a consideration of the power law just for the wrist condition (see Fig. 3) indicates that the difference in average speeds for joints would not explain the data (although variations in range of motion might). The power law plots for the wrist condition resemble fans rather than two parts of the same linear function.

The issue here is that of motor constancy. There are a variety of ways of performing curved drawing movements. Solutions are just simpler over simple joints than complex joints. Indeed higher order principles may be required for motion over more complex joints as there are potentially separate power laws that can operate for each joint.

Given that the power law applies best to simple joints, and less so for complex joints, this implies that curved drawing motion is more akin to an improvisational performance where individual joints (with their properties) are co-opted on an ad hoc basis (Schmidt, 1975). Provided that the joints/segments mesh together well, the performance occurs smoothly. This may be the case where one joint can dominate the movement endpoint outcome. Where there is greater complexity in the joints involved in movement completion, performance can be compromised and there appears to be a greater computational load. This is particularly intriguing given that the power law is poorest in situations where humans are more dextrous.

3.4. Constraint and coordination

Redundant degrees of freedom would be particularly problematic during the early stages of skill acquisition as excess degrees of freedom are thought to compromise the individual's ability to acquire the early features of a skill (Berthouze & Lungarella, 2004). Bernstein (1967) proposed that the process of skill acquisition occurs in a number of stages which alternate between freeing and freezing of degrees of freedom. The initial stages of task acquisition are associated with a locking up of degrees of freedom. As skill is acquired, degrees of freedom are once again freed up. Although there is some suggestion in the literature that Bernstein's stages of skill acquisition are not entirely borne out empirically, the notion of alternate freeing and freezing of degrees of freedom has been empirically verified in the context of skill acquisition across a range of tasks and experimental perturbations (Berthouze & Lungarella, 2004; Higuchi, Imanaka, & Hatayama, 2002). Invariances, such as the power law, also provide a solution to the degrees of freedom problem as they specify the manner in which movement will occur and hence serve to disambiguate the process of task completion. In the case of the wrist condition, there is a greater tendency to vary, and possibly greater freeing of degrees of freedom in the task where people should be more dextrous.

3.5. A computational continuum

From a computational viewpoint the power law would appear to be a demonstration of a situation where the coordinative system takes advantage of assumed relationships to reduce on-line computational loads. In this case, it appears that the system recognises the properties of muscles around a joint when performing faster curved movements. The power law represents a situation in which there are assumed relationships relied upon by the motor system to reduce its computational load. The parameters which are assumed and hence not calculated include arm length and gravity. Since the power law does not appear to rely exclusively on computation, it can be conceptualised as representing, at least to some degree, an assumed phenomenon or an automatic process. However, the observation that the power law can be modulated suggests that there is an element of this phenomenon which is controlled (Saling & Phillips, 2002). This suggests that the power law is not a wholly self-organising process.

Although the computational and dynamical accounts of the power law tend to be framed in dichotomous terms, it does appear that the empirical observations to date lend support for both accounts. As such, the mechanisms underlying invariances, such as the power law, appear to have both self-organising and computational elements. If so, the power law as a phenomenon could be considered to fall on a continuum somewhere between the extremes of computational and dynamical/ecological accounts of motion.

An alternative conceptualisation of the power law is that some aspects of this phenomenon are controlled and others are self-organised. It could be the case that where the joint system is more complex (e.g., wrist), the power law is more a product of computation than when the joint system is simple (e.g., finger).

Certainly, the complexity of the power law does not seem to be captured by the current dichotomous approach. Although these aspects are theoretically independent, they are not independent in a practical sense (see, e.g., Newell, 1973). Constructs may be fully independent under extreme conditions, such as laboratory situations or neurological disconnection syndromes, but this independence does not capture real world complexity. In the case of the power law, this approach prevents a complete understanding of a highly complicated phenomenon and results in oversimplification.

Acknowledgment

The authors would like to acknowledge Rowan Ogeil for assistance with data analysis.

References

Bernstein, N. (1967). *The coordination and regulation of movement*. London: Pergamon.

Berthouze, L., & Lungarella, M. (2004). Motor skill acquisition under environmental perturbations: On the necessity of alternate freezing and freeing of degrees of freedom. *Adaptive Behavior, 12*, 47–64.

Flash, T., & Hogan, N. (1985). The coordination of arm movements: An experimentally confirmed mathematical model. *The Journal of Neuroscience, 5*, 1688–1703.

Flash, T., & Sejnowski, T. J. (2001). Computational approaches to motor control. *Current Opinions in Neurobiology, 11*, 655–662.

Georgopolous, A. P., Kettner, R. E., & Schwartz, A. B. (1988). Primate motor cortex and free arm movements to visual targets in three dimensional space. II. Coding of the direction of movement by a neuronal population. *Journal of Neuroscience, 8*, 2928–2937.

Gribble, P. L., & Ostry, D. L. (1996). Origins of the power law relation between movement velocity and curvature: Modeling the effects of muscle mechanics and limb dynamics. *Journal of Neurophysiology, 76*, 2853–2860.

Higuchi, T., Imanaka, K., & Hatayama, T. (2002). Freezing degrees of freedom under stress: Kinematic evidence of constrained movement strategies. *Human Movement Science, 21*, 831–846.

Ito, L., Kawakami, Y., Ichinose, Y., Fukahiro, S., & Fukunaga, T. (1998). Nonisometric behavior of fascicles during isometric contractions of a human muscle. *Journal of Applied Physiology, 85*, 1230–1235.

Ivanenko, Y. P., Grasso, R., Macellari, V., & Lacquaniti, F. (2002). Two-thirds power law in human locomotion: Role of ground contact forces. *Neuroreport: For Rapid Communication of Neuroscience Research, 13*, 1171–1174.

Kelso, J. A. S. (1995). *Dynamic patterns: The self-organisation of brain and behavior*. Cambridge, MA: MIT.

Lacquaniti, F., Terzuolo, C., & Viviani, P. (1983). The law relating the kinematic and figural aspects of drawing movements. *Acta Psychologica, 54*, 115–130.

Lacquaniti, F., Terzuolo, C., & Viviani, P. (1984). Global metric properties and preparatory processes in drawing movements. In S. Kornblum & J. Requin (Eds.), *Preparatory processes in drawing movements* (pp. 357–370). Hillsdale, NJ: Lawrence Erlbaum.

Massey, J. T., Lurito, J. T., Pellizer, G., & Georgopoulos, A. P. (1992). Three Dimensional drawings in isometric conditions: Relation between geometry and kinematics. *Experimental Brain Research, 88*, 685–690.

Newell, A. (1973). You can't play 20 questions with nature and win. In W. G. Chase (Ed.), *Visual information processing* (pp. 283–308). New York: Academic.

Pellizer, G., Massey, J. T., Lurito, J. T., & Georgopolous, A. P. (1992). Three- dimensional drawings in isometric conditions: Planar segmentation of force trajectory. *Experimental Brain Research, 92*, 326–337.

Saling, L. L., & Phillips, J. G. (2002). Age-related changes in the kinematics of curved drawing movements: Relationship between tangential velocity and the radius of curvature. *Experimental Aging Research, 28*, 215–229.

Schaal, S., & Sternad, D. (2001). Origins and violations of the 2/3 power law in rhythmic three-dimensional arm movements. *Experimental Brain Research, 136*, 60–72.

Schmidt, R. A. (1975). A schema theory of discrete motor skill learning. *Psychological Review, 82*, 225–260.

Schmidt, R. A. (1984). The search for invariance in skilled movement behavior. *Research Quarterly for Exercise and Sport, 56*, 188–200.

Sciaky, R., Lacquaniti, F., & Terzuolo, C. (1987). A note on the kinematics of drawing movements in children. *Journal of Motor Behavior, 19*, 518–525.

Tasko, S. M., & Westbury, J. R. (2004). Speed–curvature relations for speech-related articulatory movement. *Journal of Phonetics, 32*, 65–80.

Teulings, H.-L., Thomassen, A. J. W. M., & Maarse, F. J. (1989). A description of handwriting in terms of main axes. In R. Plamondon, C. Y. Suen, & M. L. Simner (Eds.), *Computer recognition and human production of handwriting* (pp. 193–211). Singapore: World Scientific.

Todorov, E., & Jordan, M. I. (1998). Smoothness maximization along a predefined path accurately predicts the speed profiles of complex arm movements. *Journal of Neurophysiology, 80*, 696–714.

Viviani, P., & Flash, T. (1995). Minimum-jerk, two-thirds power law, and isochrony: Converging approaches to movement planning. *Journal of Experimental Psychology, 21*, 32–53.

Viviani, P., & Schneider, W. (1991). A developmental study of the relationship between geometry and kinematics in drawing movements. *Journal of Experimental Psychology: Human Perception and Performance, 17*, 198–218.

Wann, J., Nimmo-Smith, I., & Wing, A. M. (1988). Relation between velocity and curvature in movement: Equivalence and divergence between a power law and minimum jerk model. *Journal of Experimental Psychology: Human Perception and Performance, 14*, 622–637.